## Lecture 21

CSE 331
Oct 21, 2016

## Grading

Mid-term-1 hopefully by today

Mini project pitch by the weekend

## Scheduling to minimize lateness

n jobs: ith job $\left(\mathrm{t}_{\mathrm{i}}, \mathrm{d}_{\mathrm{i}}\right)$
start time: s
Schedule the $n$ jobs: ith job gets interval $\left[s(i), f(i)=s(i)+t_{i}\right)$

Algo picks s(i)
GOAL: Minimize MAXIMUM lateness

Lateness of job $\mathrm{i}, \mathrm{I}_{\mathrm{i}}=\max \left(0, \mathrm{f}(\mathrm{i})-\mathrm{d}_{\mathrm{i}}\right)$

## The Greedy Algorithm

(Assume jobs sorted by deadline: $\mathrm{d}_{1} \leq \mathrm{d}_{2} \leq \ldots . . \leq \mathrm{d}_{\mathrm{n}}$ )

$$
f=s
$$

For every i in 1..n do
Schedule job i from $s(i)=f$ to $f(i)=f+t_{i}$ $f=f+t_{i}$

## Two definitions for schedules

Idle time Max "gap" between two consecutively scheduled tasks


Inversion
$(i, j)$ is an inversion if $i$ is scheduled before $j$ but $d_{i}>d_{j}$


For every i in 1..n do
Schedule job ifrom $s_{i}=f$ to $f_{i}=f+t_{i}$

## Proof structure

Any two schedules with 0 idle time and 0 inversions have the same max lateness

$$
\text { Greedy schedule has } 0 \text { idle time and } 0 \text { inversions }
$$

There is an optimal schedule with 0 idle time and 0 inversions

## Today's agenda

"Exchange" argument to convert an optimal solution into a 0 inversion one

## Rest of Today

## Shortest Path Problem



## Reading Assignment

Sec 2.5 of [KT]


## Shortest Path problem

Input: Directed graph $G=(\mathrm{V}, \mathrm{E})$
Edge lengths, $I_{e}$ for $e$ in $E$

"start" vertex s in V


Output: All shortest paths from s to all nodes in V

# Naïve Algorithm 

$\Omega(\mathrm{n}!)$ time

## Dijkstra' s shortest path algorithm



