#### Lecture 37

CSE 331 Dec 2, 2016

# Quiz 2 on Monday



#### You can use two letter sized cheatsheets

## Last HW up!

## Homework 10

Due by 12:30pm, Friday, December 9, 2016.

Make sure you follow all the homework policies.

All submissions should be done via Autolab.

#### Question 1 (Programming Assignment) [40 points]

#### Note

This assignment can be solved in either Java, Python or C++ (you should pick the language you are most comfortable with). Please make sure to look at the supporting documentation and files for the language of your choosing.

#### The Problem

In this problem, you are given a directed graph (in adjacency list representation) G = (V, E) where each each edge  $e \in E$  has cost  $c_e$  (which can be negative but G does

## HW 9 solutions

#### At the END of the lecture

## Shortest Path Problem

Input: (Directed) Graph G=(V,E) and for every edge e has a cost  $c_e$  (can be <0)

t in V

Output: Shortest path from every s to t





#### When to use Dynamic Programming



There are polynomially many sub-problems

**Richard Bellman** 

Optimal solution can be computed from solutions to sub-problems

There is an ordering among sub-problem that allows for iterative solution

## Sub-problems

OPT(u,i) = cost of shortest path from u to t with at most i edges

#### When to use Dynamic Programming



There are polynomially many sub-problems

**Richard Bellman** 

Optimal solution can be computed from solutions to sub-problems

There is an ordering among sub-problem that allows for iterative solution

# Today's agenda

Finish Bellman-Ford algorithm

Analyze the run time

#### Natural order among OPT(u,i) values?

## The recurrence

OPT(u,i) = shortest path from u to t with at most i edges

 $OPT(u,i) = \min \left\{ OPT(u,i-1), \min_{(u,w) \text{ in } E} \left\{ c_{u,w} + OPT(w,i-1) \right\} \right\}$ 

#### Some consequences

OPT(u,i) = shortest path from u to t with at most i edges

$$OPT(u,i) = \min \left\{ OPT(u, i-1), \min_{(u,w) \text{ in } E} \left\{ c_{u,w} + OPT(w,i-1) \right\} \right\}$$

OPT(u,n-1) is shortest path cost between u and t

Group talk time: How to compute the shortest path between **s** and **t** given all OPT(u,i) values

### Longest path problem

Given G, does there exist a simple path of length n-1 ?

#### Longest vs Shortest Paths



# Two sides of the "same" coin

Shortest Path problem

Can be solved by a polynomial time algorithm

Is there a longest path of length n-1?



Given a path can verify in polynomial time if the answer is yes

# Poly time algo for longest path?





#### **Clay Mathematics Institute**

Dedicated to increasing and disseminating mathematical knowledge

HOHE ABOUT CHI PROGRAME NEWS & EVENTS AWARDS SCHOLARS PUBLICATIONS

#### First Clay Mathematics Institute Millennium Prize Announced

Prize for Resolution of the Poincaré Conjecture Awarded to Dr. Grigoriy Perelman

- \* Birch and Swinnerton-Dver Conjecture
- \* Hodge Conjecture
- \* Navier-Stokes Equations
- P vs NP
- Polincaré Conjecture
- to Philadel and a second build and a second second

#### P vs NP question

 $\mathbf{P}$ : problems that can be solved by poly time algorithms



NP: problems that have polynomial time verifiable witness to optimal solution

Alternate NP definition: Guess witness and verify!

### Proving $P \neq NP$

Pick any one problem in NP and show it cannot be solved in poly time

Pretty much all known proof techniques *provably* will not work

# Proving P = NP

Will make cryptography collapse

Compute the encryption key!

Prove that all problems in NP can be solved by polynomial time algorithms



## A book on P vs. NP



# High level view of CSE 331



## If you are curious for more

CSE 429 or 431: Algorithms

CSE 396: Theory of Computation

