Lecture 6

CSE 331

Sep 12, 2016

Mini project group due in 2 weeks

CSE 331 Mini project choices

Fall 2016

Please check the table below before submitting your mini project pitch to make sure your case study is not being used by another group. Case studies are assigned on a first come first serve basis.

Group	Societal Aspect	Case Studies
Anand Balakrishnan, Vikram Garu and Veronica Ng		
Hank Lin, Michael Tobio and Miaomiao Zhang		
Devashish Agarwal, Jacob Fijas and Kevin Rathbun		
Sravanika Doddi, Anne Izydorczak and Simran Singh		
Vighnesh Iyer, Nicholas LaGrassa and Kartikeya Shukla		

Gale-Shapley Algorithm

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Output the engaged pairs S as the final output

Observation 1

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Once a man gets engaged, he remains engaged (to "better" women)

Output the engaged pairs S as the final output

Observation 2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

If w proposes to m after m', then she prefers m' to m

Output the engaged pairs S as the final output

Proof via "progress"

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

End of iteration t, define progress P(t) s.t.:

1. $1 \leq P(t) \leq n^2$
2. $P(t+1) = P(t)+1$

Output the engaged pairs as the final output

Today's lecture

GS algorithms always outputs a stable marriage

The Lemmas

Lemma 1: The GS algorithm has at most n^2 iterations

Lemma 2: S is a perfect matching

Lemma 3: S has no instability

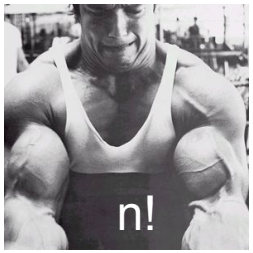
Questions/Comments?

Extensions

Fairness of the GS algorithm

Different executions of the GS algorithm

Main Steps in Algorithm Design



Problem Statement

Problem Definition

Algorithm

"Implementation"

Analysis

Correctness Analysis

Definition of Efficiency

An algorithm is efficient if, when implemented, it runs quickly on real instances

Implemented where?

Platform independent definition

What are real instances?

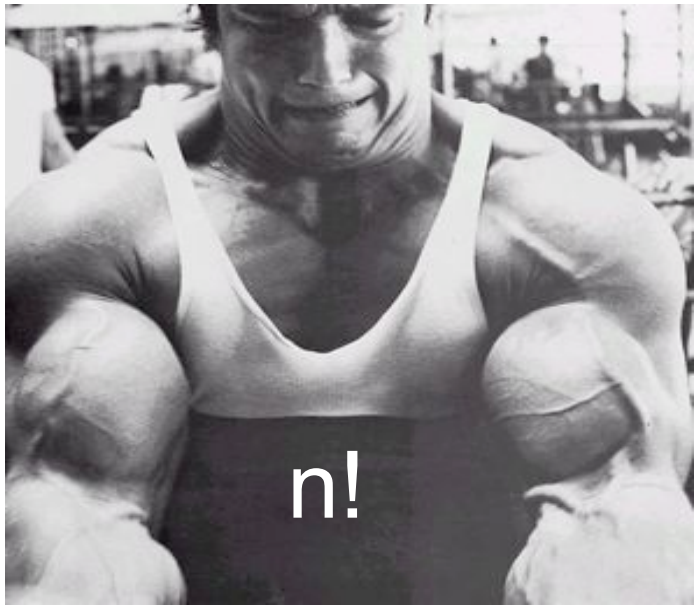
Worst-case Inputs

Efficient in terms of what?

$N = 2n^2$ for SMP

Input size N

Definition-II



Analytically better than brute force

How much better? By a factor of 2?

Definition-III

Should scale with input size

If N increases by a constant factor, so should the measure

Polynomial running time

At most $c \cdot N^d$ steps ($c>0$, $d>0$ absolute constants)

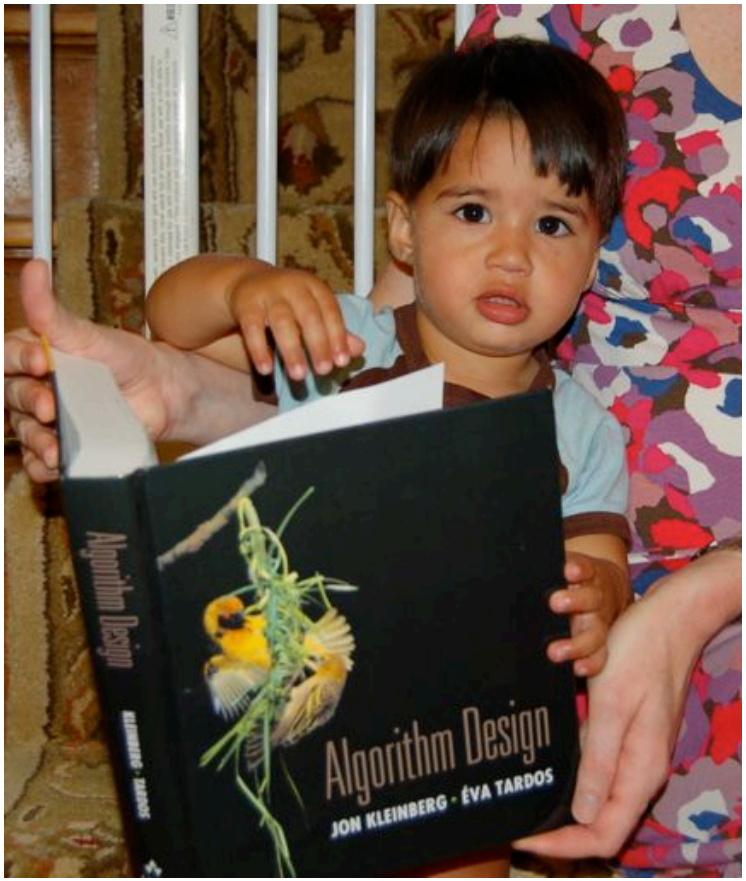
Step: “primitive computational step”

More on polynomial time

Problem centric tractability

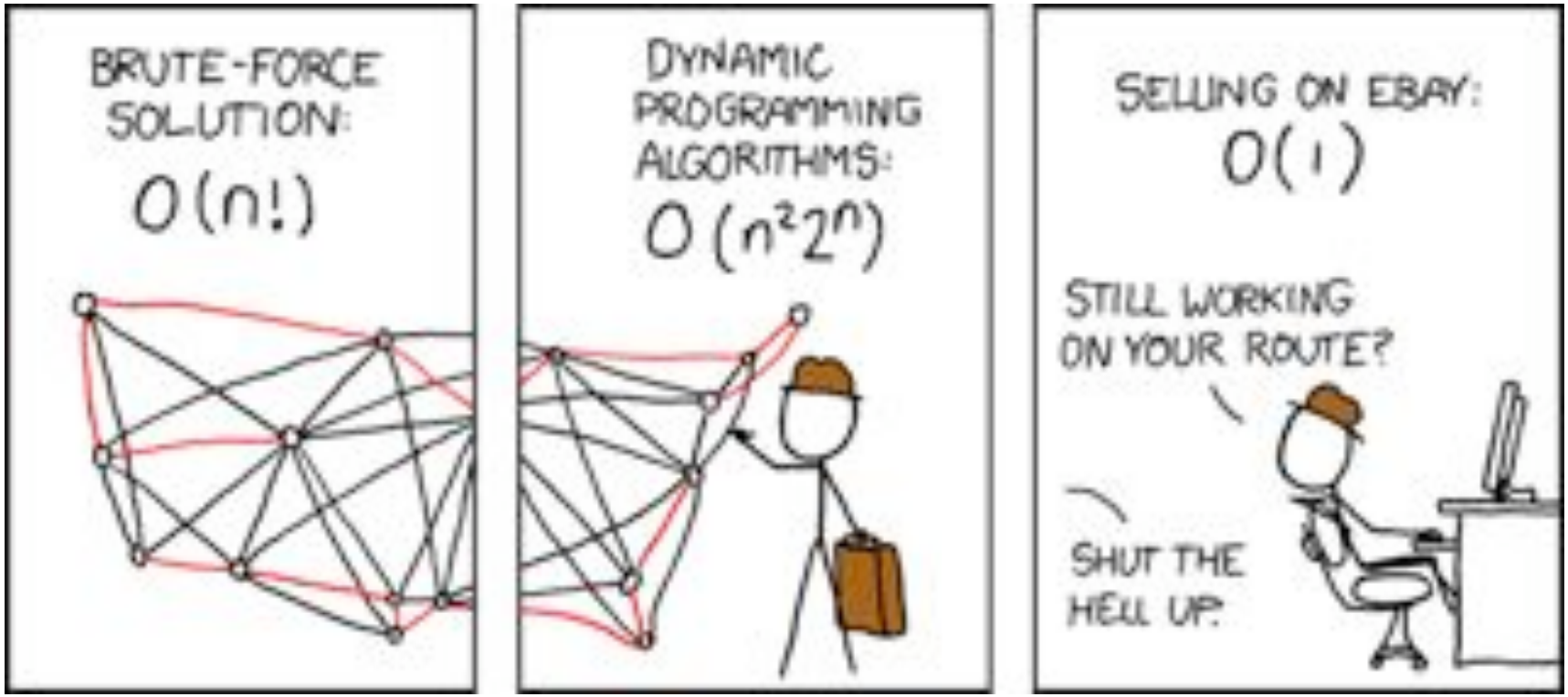
Can talk about problems that are not efficient!

Reading Assignments



Sections 1.2, 2.1, 2.2 and 2.4 in [KT]

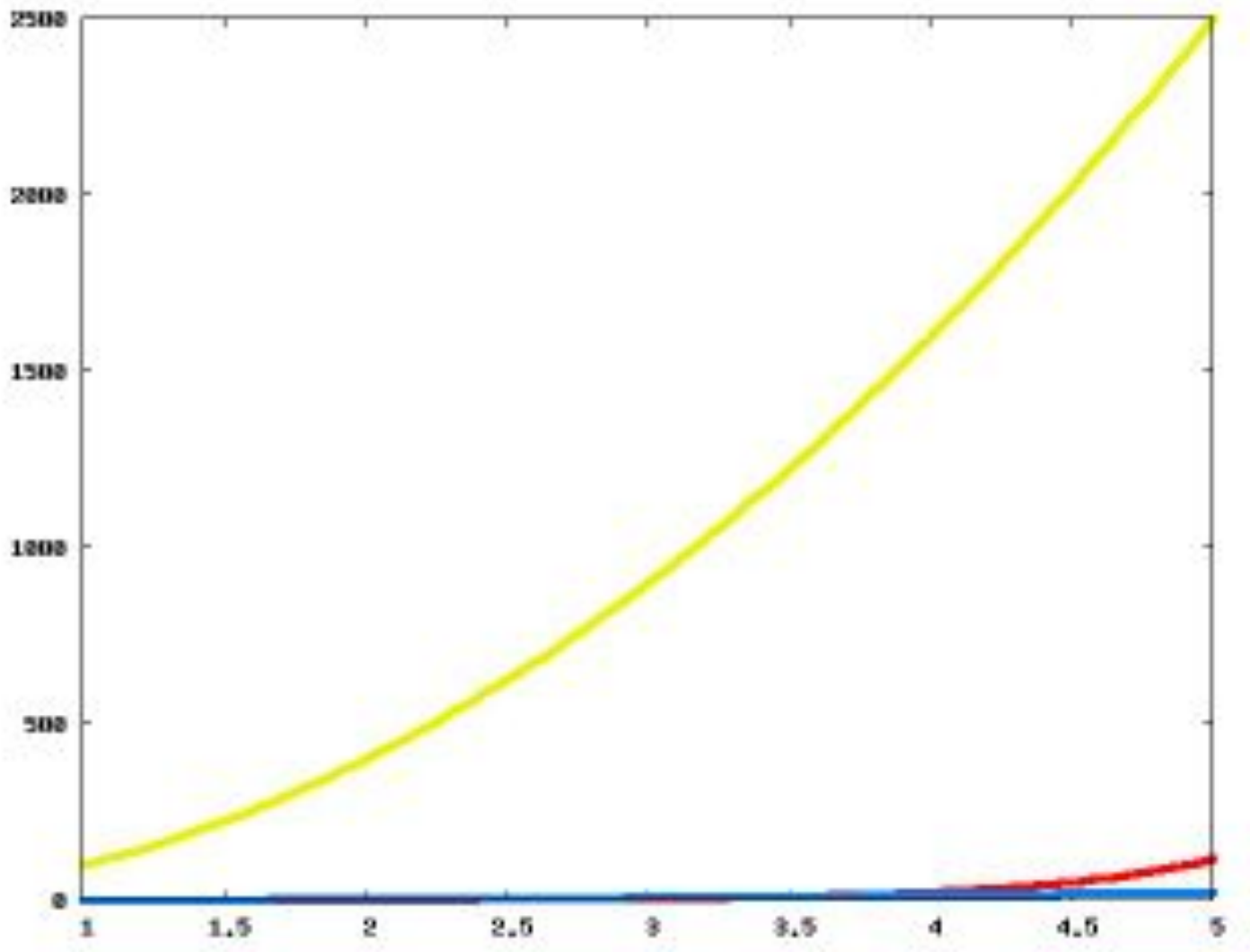
Asymptotic Analysis



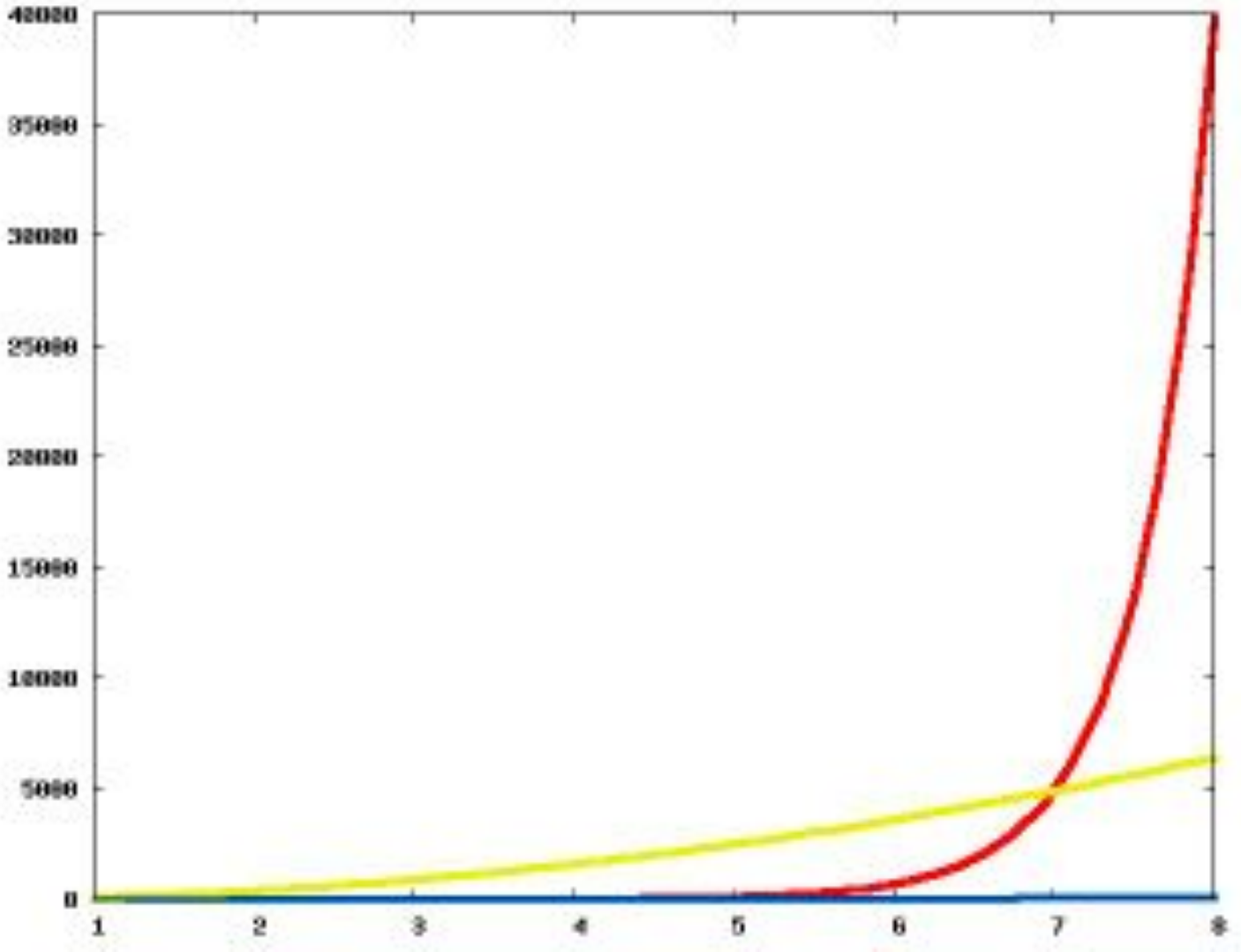
Travelling Salesman Problem

(http://xkcd.com/399/)

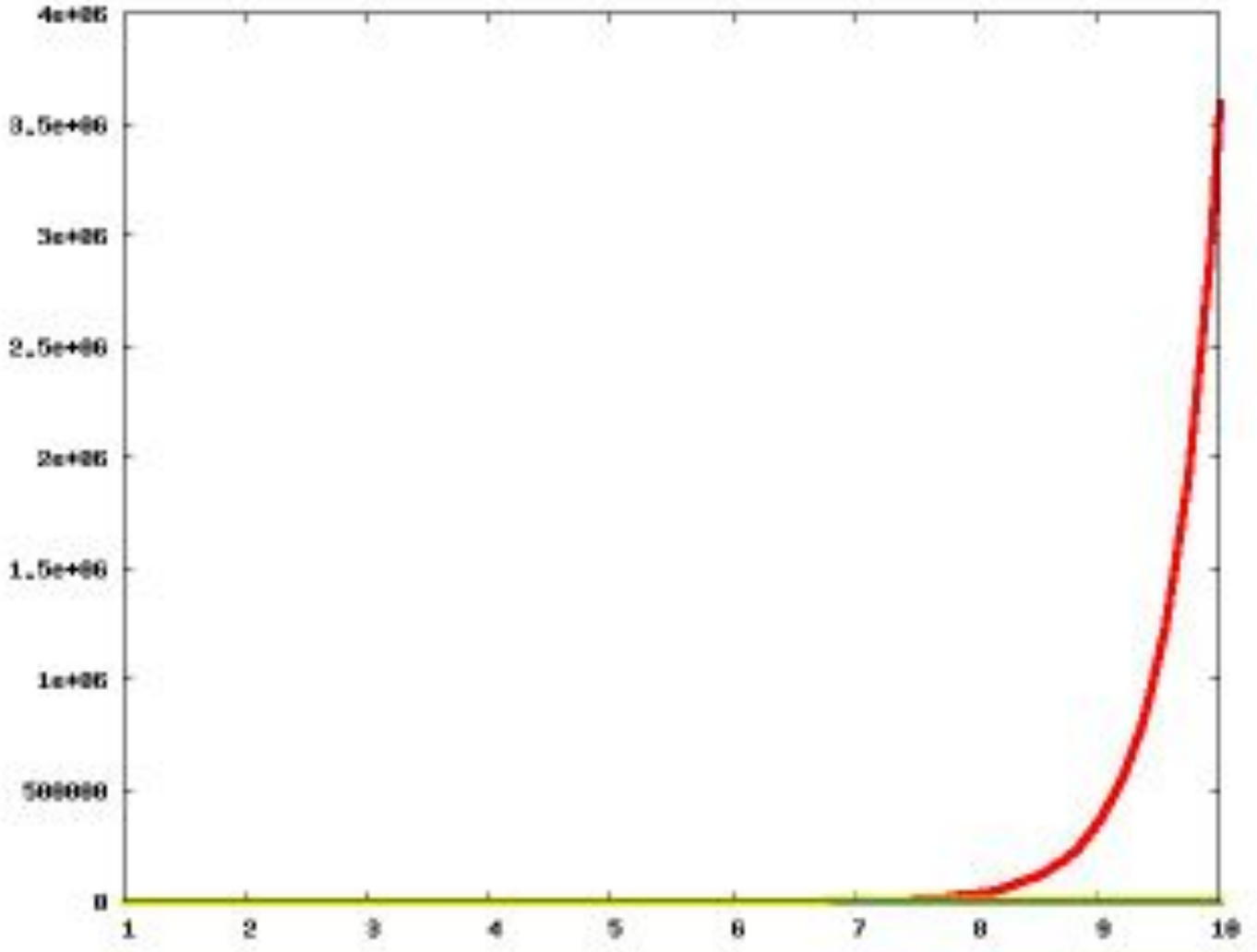
Which one is better?



Now?



And now?

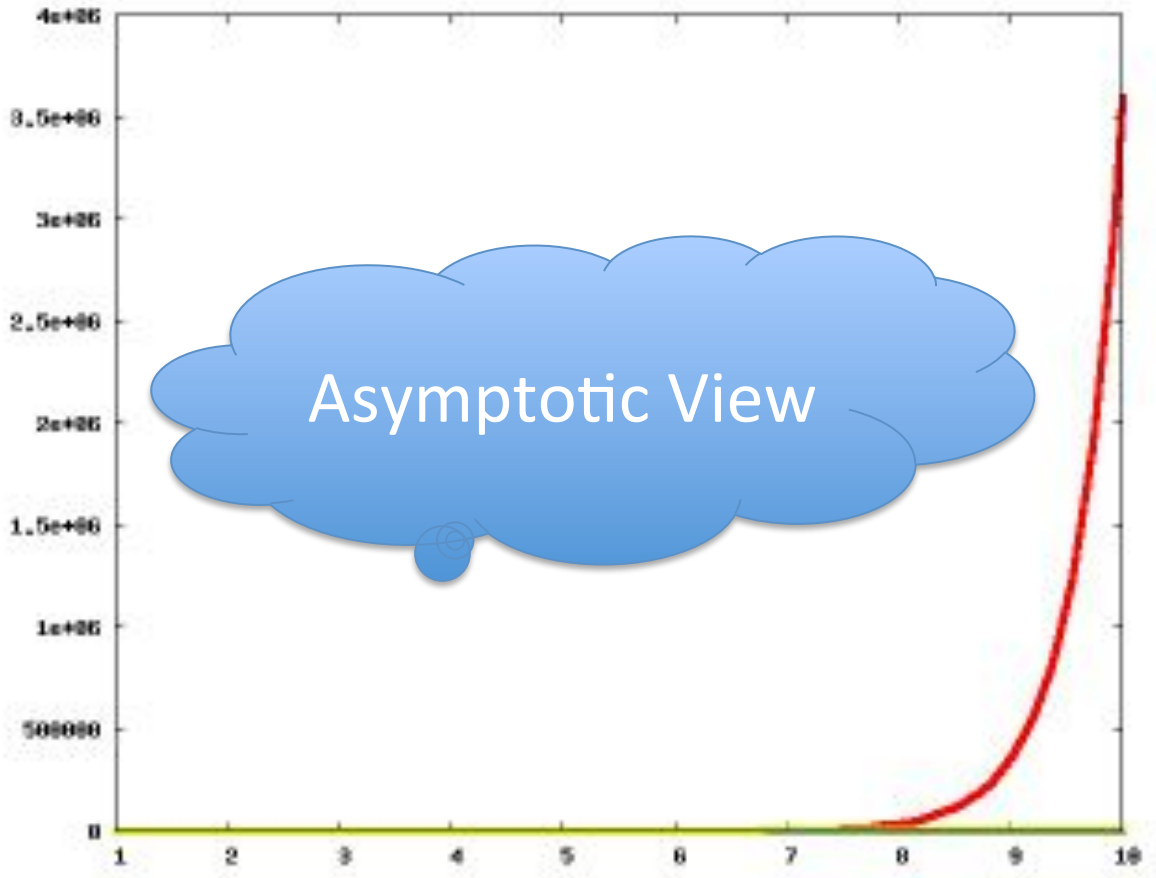


The actual run times

n!

$100n^2$

n^2



Asymptotic Notation

≤ is O with glasses

≥ is Ω with glasses

= is Θ with glasses