

# Lecture 8

CSE 331

Sep 16, 2016

# HW 2 has been posted

## Homework 2

Due by **12:30pm, Friday, September 23, 2016.**

Make sure you follow all the [homework policies](#).

All submissions should be done via [Autolab](#).

## Sample Problem

### The Problem

This problem is just to get you thinking about asymptotic analysis and input sizes.

An integer  $n \geq 2$  is a prime, if the only divisors it has is 1 and  $n$ . Consider the following algorithm to check if the given number  $n$  is prime or not:

For every integer  $2 \leq i \leq \sqrt{n}$ , check if  $i$  divides  $n$ . If so declare  $n$  to be not a prime. If no such  $i$  exists, declare  $n$  to be a prime.

What is the function  $f(n)$  such that the algorithm above has running time  $\Theta(f(n))$ ? Is this a polynomial running time-- justify your answer. (A tangential question: Why is the algorithm correct?)

# Autolab Updates

## Q1 (NRMP)

### Admin Options

Edit assessment

Grade submissions

Release all grades

Withdraw all grades

Export assessment

Reload config file

Manage extensions

Manage submissions

View statistics

Bulk import grades

Bulk export grades

*Submit your solution in any of C++/Java/Python. Note that autograding feedback will stop at Thursday midnight (though you can still submit till Friday 12:30pm but you will only see a grade of 0.)*

**Due at:** Friday, Sep 23rd 2016, 12:30:00 pm

**Last day to handin:** Friday, Sep 23rd 2016, 12:30:00 pm

**Language \*:**

**Sources \*:**

\* denotes required fields. The submission cannot be completed without filling out the required fields.

Submit File

# Autolab Updates

## Q2 (Home wrecker)

### Admin Options

Edit assessment

Grade submissions

Release all grades

Withdraw all grades

Export assessment

Reload config file

Manage extensions

Manage submissions

View statistics

Bulk import grades

Bulk export grades

*Remember to make sure you preview your uploaded PDF to make sure it is not corrupted. If you did not use any sources or collaborate just say "None" in the appropriate textbox.*

**Due at:** Friday, Sep 23rd 2016, 12:21:00 pm

**Last day to handin:** Friday, Sep 23rd 2016, 12:30:00 pm

**Sources \*:**

**Collaborators \*:**

\* denotes required fields. The submission cannot be completed without filling out the required fields.

Submit File

# Solutions to HW1

Handed out at the end of the lecture

# Project group due in a week

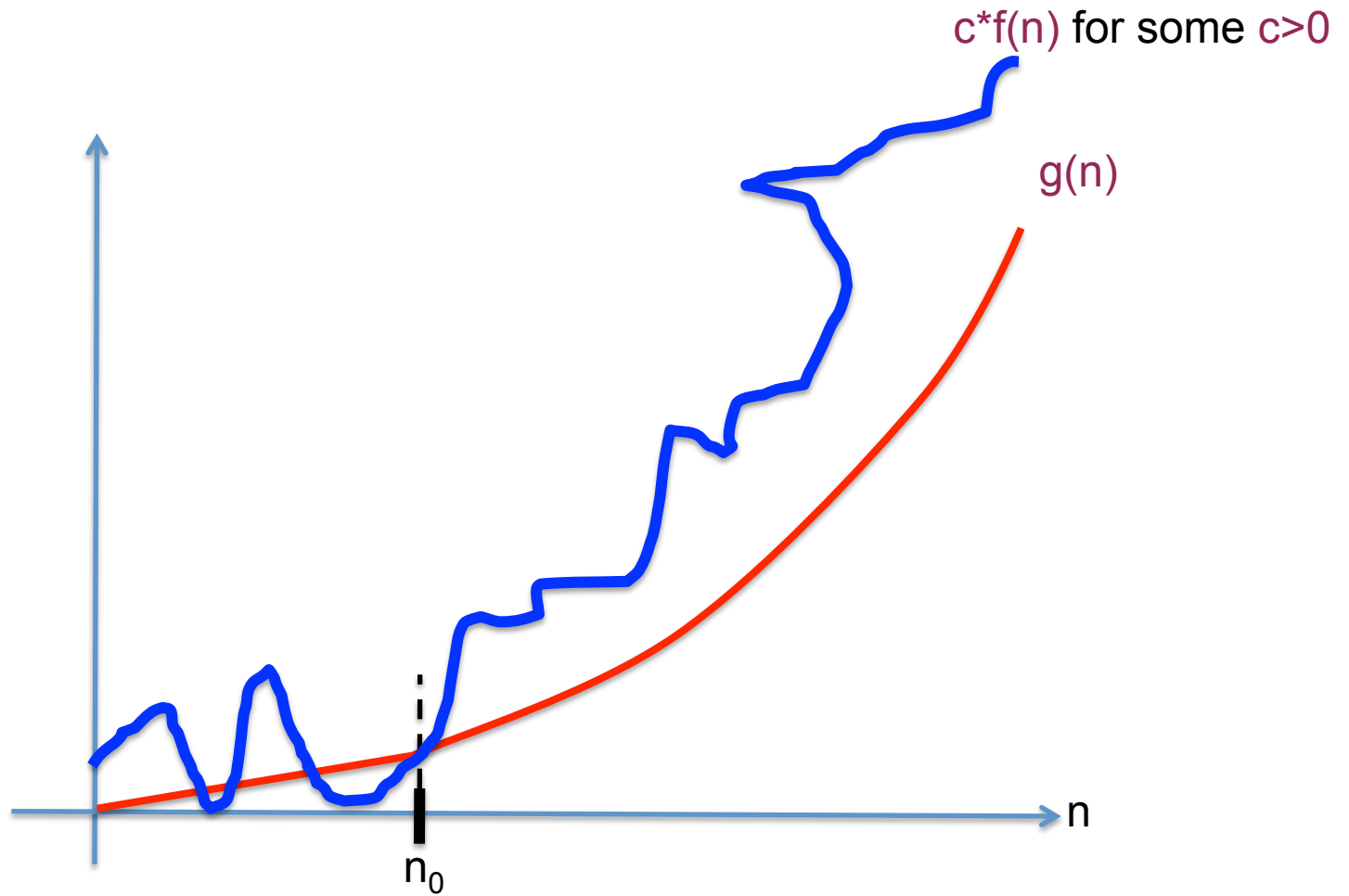
## CSE 331 Mini project choices

Fall 2016

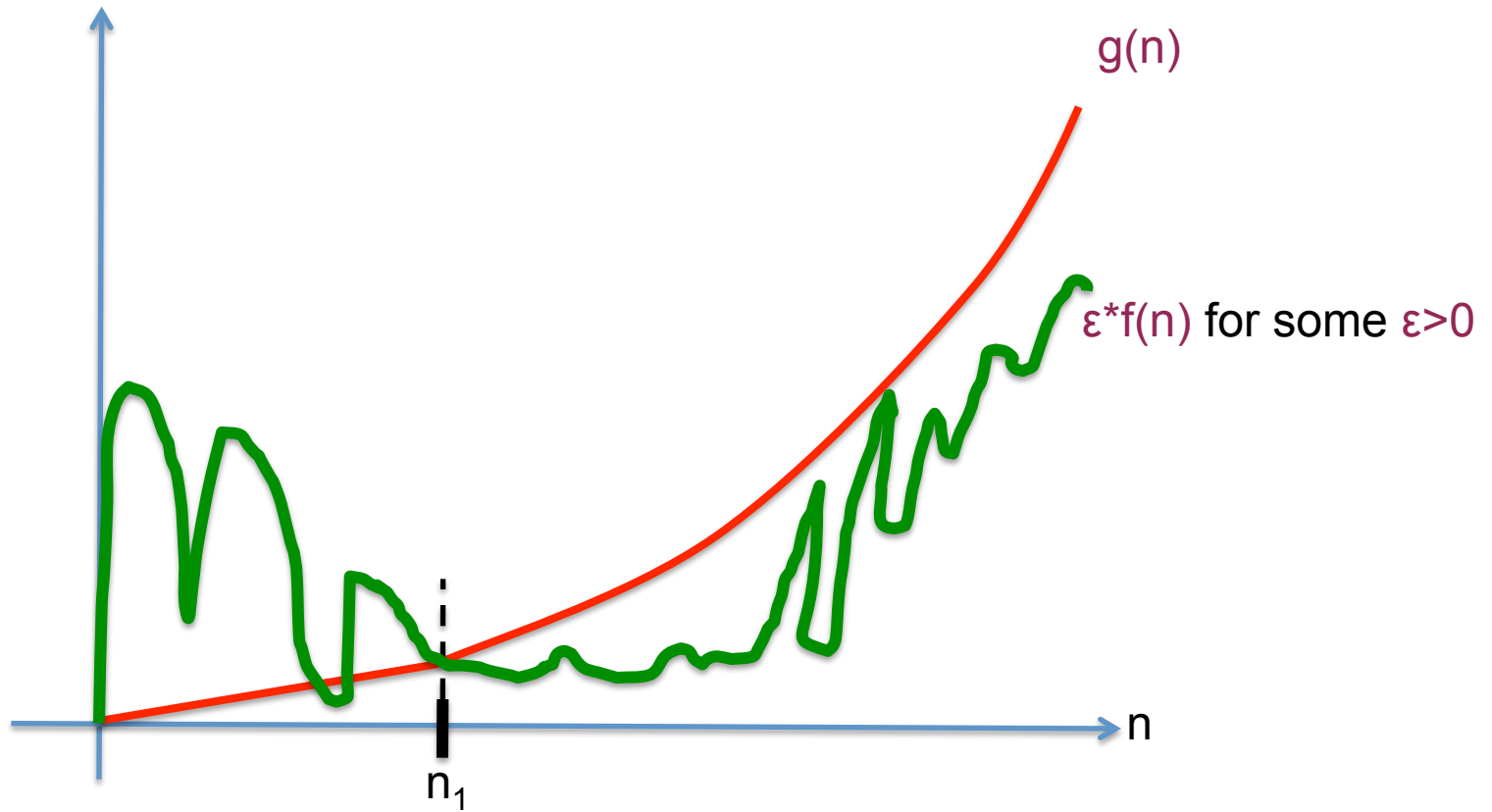
Please check the table below before submitting your mini project pitch to make sure your case study is not being used by another group. Case studies are assigned on a first come first serve basis.

Group	Societal Aspect	Case Studies
Anand Balakrishnan, Vikram Garu and Veronica Ng		
Hank Lin, Michael Tobio and Miaomiao Zhang		
Devashish Agarwal, Jacob Fijas and Kevin Rathbun		
Sravanika Doddi, Anne Izydorczak and Simran Singh		
Vignesh Iyer, Nicholas LaGrassa and Kartikeya Shukla		
William Burgin, Aakanksha Raike and Andrea Schmidin		

$g(n)$  is  $O(f(n))$



$g(n)$  is  $\Omega(f(n))$





# Properties of $O$ (and $\Omega$ )

Transitive

$g$  is  $O(f)$  and  $f$  is  $O(h)$  then  
 $g$  is  $O(h)$

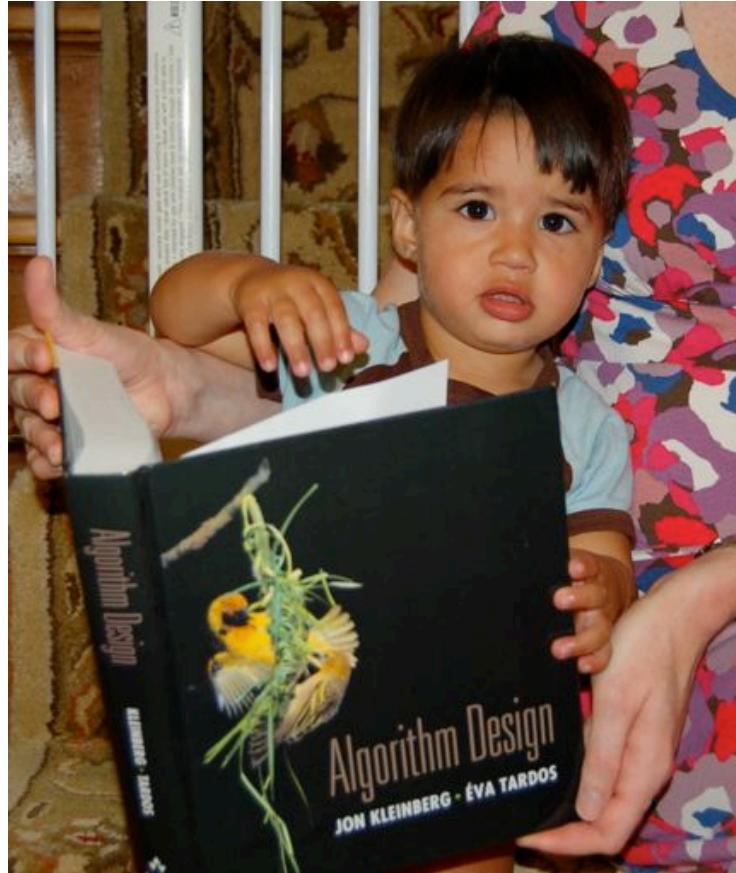
Additive

$g$  is  $O(h)$  and  $f$  is  $O(h)$  then  
 $g+f$  is  $O(h)$

Multiplicative

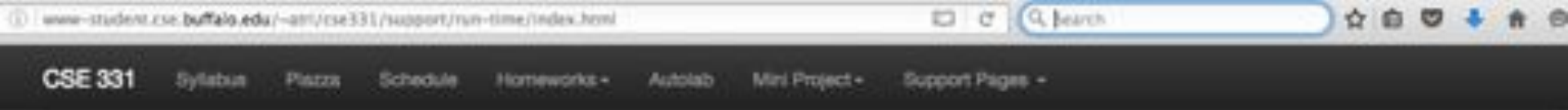
$g$  is  $O(h_1)$  and  $f$  is  $O(h_2)$  then  
 $g*f$  is  $O(h_1*h_2)$

# Reading Assignments



Sections 1.1, 1.2, 2.1, 2.2 and 2.4 in [KT]

# Another Reading Assignment



## Analyzing the worst-case runtime of an algorithm

Some notes on strategies to prove Big-Oh and Big-Omega bounds on runtime of an algorithm.

### The setup

Let  $\mathcal{A}$  be the algorithm we are trying to analyze. Then we will define  $T(N)$  to be the worst-case run-time of  $\mathcal{A}$  over all inputs of size  $N$ . Slightly more formally, let  $t_{\mathcal{A}}(\mathbf{x})$  be the number of steps taken by the algorithm  $\mathcal{A}$  on input  $\mathbf{x}$ . Then

$$T(N) = \max_{\mathbf{x} \text{ is of size } N} t_{\mathcal{A}}(\mathbf{x}).$$

In this note, we present two useful strategies to prove statements like  $T(N)$  is  $O(g(N))$  or  $T(N)$  is  $\Omega(h(N))$ . Then we will analyze the run time of a very simple algorithm.

### Preliminaries

We now collect two properties of asymptotic notation that we will need in this note (we saw these in class today).

# Questions?



# Today's agenda

Asymptotic run time

Analyzing the run time of the GS algo

# Gale-Shapley Algorithm

Initially all men and women are **free**

While there exists a free woman who can propose

Let  $w$  be such a woman and  $m$  be the best man she has not proposed to

$w$  proposes to  $m$

If  $m$  is free

$(m,w)$  get **engaged**

Else  $(m,w')$  are engaged

If  $m$  prefers  $w'$  to  $w$

$w$  remains **free**

Else

$(m,w)$  get **engaged** and  $w'$  is **free**

Output the engaged pairs as the final output

# Implementation Steps

How do we represent the input?

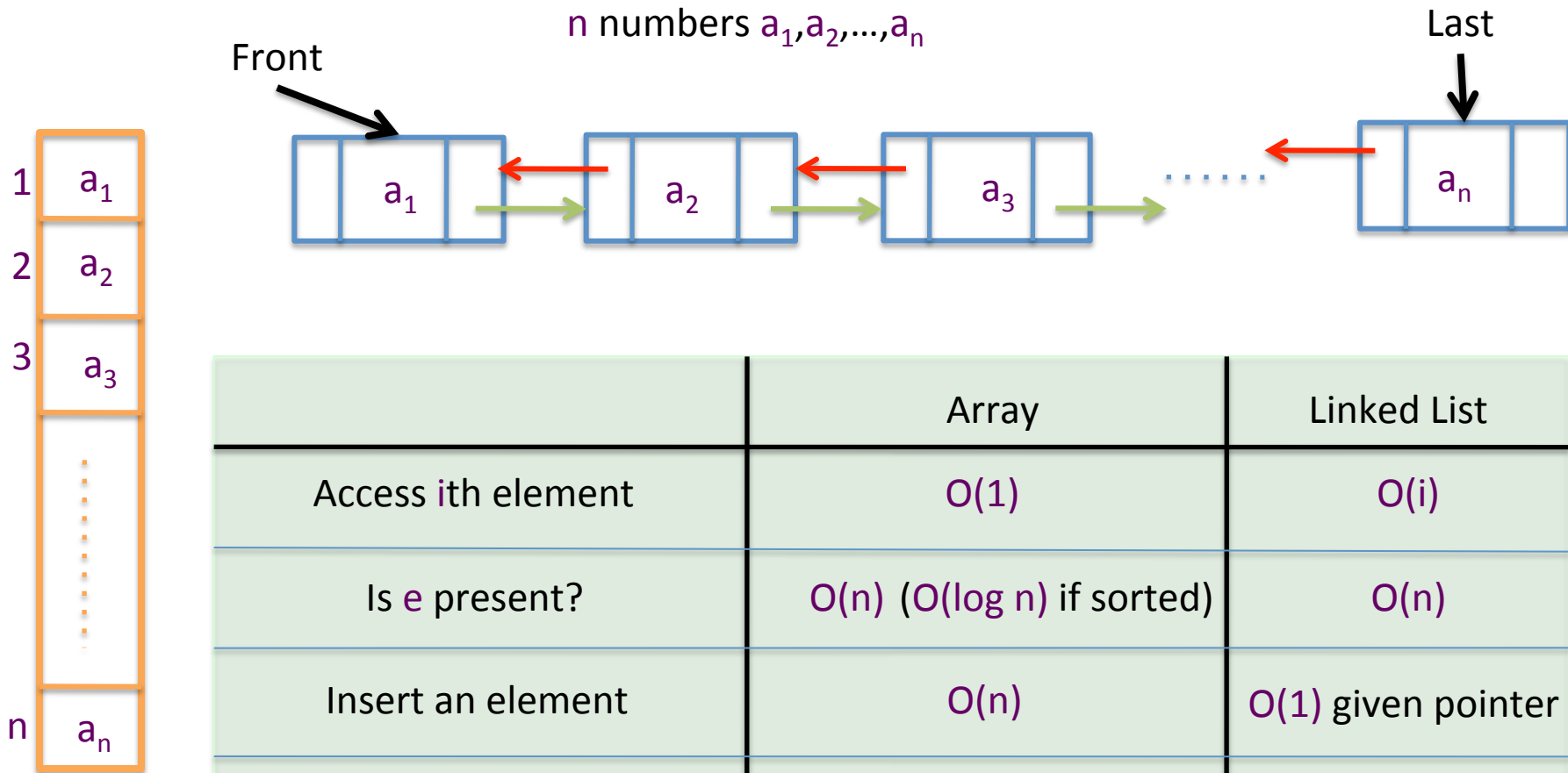
How do we find a free woman  $w$ ?

How would  $w$  pick her best unproposed man  $m$ ?

How do we know who  $m$  is engaged to?

How do we decide if  $m$  prefers  $w'$  to  $w$ ?

# Arrays and Linked Lists



	Array	Linked List
Access $i$ th element	$O(1)$	$O(i)$
Is $e$ present?	$O(n)$ ( $O(\log n)$ if sorted)	$O(n)$
Insert an element	$O(n)$	$O(1)$ given pointer
Delete an element	$O(n)$	$O(1)$ given pointer
Static vs Dynamic	Static	Dynamic



# Today's agenda

$O(n^2)$  implementation of the Gale-Shapley algorithm

More practice with run time analysis



# Gale-Shapley Algorithm

Initially all men and women are **free**

At most  $n^2$  iterations

While there exists a free woman who can propose

Let  $w$  be such a woman and  $m$  be the best man she has not proposed to

$w$  proposes to  $m$

If  $m$  is free

$(m,w)$  get **engaged**

Else  $(m,w')$  are engaged

If  $m$  prefers  $w'$  to  $w$

$w$  remains **free**

Else

$(m,w)$  get **engaged** and  $w'$  is **free**

$O(1)$  time  
implementation

Output the engaged pairs as the final output