Sep 21

$G = (V, E)$ $\qquad E \subseteq V \times V$

V: set of vertices / nodes; E: set of edges

<u>Default notation</u>: $n = |V|$, $m = |E|$

<u>Def</u>: A graph $G = (V, E)$ is undirected if

$(u,w) \in E \iff (w,u) \in E$

$u \rightleftarrows w$ denote this by $u - w$

G otherwise is directed

$u \rightarrow w$: $A \rightleftarrows B \rightarrow C$

directed

- (•) News host graph: Directed
- (•) Airline map: Undirected
- (•) Wikipedia graph: Directed

<u>By default</u>: A graph G is <u>undirected</u>

<u>Claim</u>: Every undirected is directed.

$u - w$ equiv $u \rightleftarrows w$

<u>Def</u>: A path in $G = (V, E)$ is a sequence of nodes

$u_1, \dots, u_k \equiv u_1\text{-}u_k$ path s.t. $\forall i < k$, $(u_i, u_{i+1}) \in E$

<u>Note</u>: u_i's can be repeated

$A - B - C - D$

- D, C, B, A ✓
- A, B, C, D ✓
- A, B, C, B ✓
- A, C, D ✗

$A \rightarrow B \rightarrow C$

- A, B, C ✓
- C, B, A ✗

<u>Def</u>: A <u>simple</u> path does not have any repeated nodes

<u>Default</u>: A path is a simple path.

Def: <u>Length</u> of a path is # of edges in it
(Distance)

Ex. D, C, B, A in  has length 3

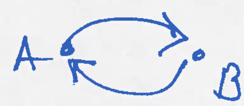
Q: What is the largest possible length of a <u>simple</u> path?

A: max length $\leq n-1$.

<u>Cycle</u>: A path $u_1, \ldots, u_k$ is a cycle if

(i) $u_1, \ldots, u_{k-1}$ are distinct

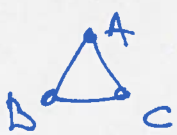
AND (ii) $u_1 = u_k$

AND (iii) Directed $k \geq 3$ 

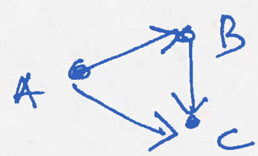
A, B, A ✓

AND (iv) Undirected $k \geq 4$ 

A, B, A ✗



A, B, C, A ✓



A, B, C, A ✗

A, B, C, A ✓

Def: $u, w \in V$ are connected, if $\exists$ exists a $u-w$ path
(recall: G is undirected by default)

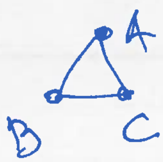
(u, w) in directed graph is <u>strongly connected</u> if $\exists$ $u \rightarrow w$ path <u>AND</u> $w \rightarrow u$ path

A, C ✓

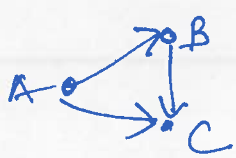
A, C strongly connected ✗

<u>Base case</u>: u is connected to u.

Def: (Directed) G is (strongly) connected if every pair of vertices in G are (strongly) connected.



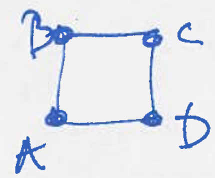
✓



✗

✓

Def: Distance between u, w in G is the **length** of the shortest u–w path.



$$\text{dist}(A, D) = \min\{3, 1\} = 1.$$