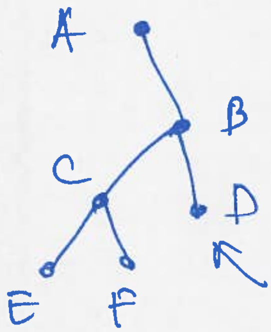
$\boxed{\text{Sep 23}}$ **Def:** An undirected graph $T=(V,E)$ is a tree if

(1) T is connected & (2) T has no cycles

~~Thm~~ **THEOREM:** A tree on n nodes has EXACTLY $n-1$ edges



$n = 6$
$m = 5$

root at B

Pf idea: Pick a root $r \in V$

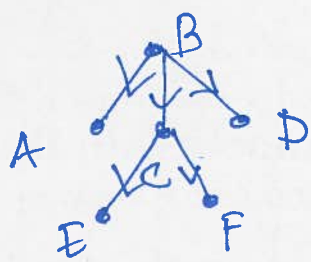
Root T at r

(1) Direct edges "away" from r

(2) Every edge has a unique child

(3) ~~Root~~ r is the only node that is not a child of any other node

parent
$\downarrow$
child



$\rightsquigarrow |E| = |V \setminus \{r\}|$

$A \setminus B = \{a \in A \mid a \notin B\}$

Pf details Pick a ~~root~~ $r \in V$ & root T at r

parent $\equiv$ tail $\to$ u closer to r

$\downarrow$

child $\equiv$ head $\to$ w ~~fartha~~ further away from r

Ex! For every $(u,w) \in E$ one of them is closer to r than the other

distance

Claim 1! r is not the head of any directed edge

Claim 2! Every directed edge has a unique head

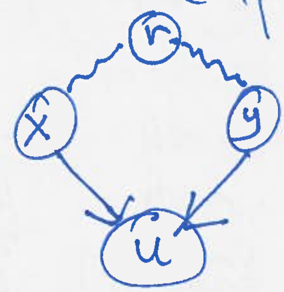
Claim 3! Every non-root vertex is the head of some edge

Claim 4! Every non-root vertex is the head of ≤ 1 edge.

Claims 1-4 $\Rightarrow$ 1-to-1 correspondence between E & $V \setminus \{r\}$

$\Rightarrow |E| = |V \setminus \{r\}| = n-1.$

Pf idea of Claim 4: For sake of contradiction assume u is a non-root that is the head of two edges

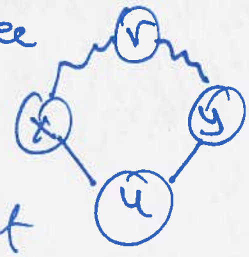


T is connected

$\Rightarrow \exists$ r-x path

& r-y path

Now consider the original tree



$\Rightarrow \exists$ a cycle in T

$\Rightarrow$ contradicts the fact that T is a tree ∎

THEOREM 2: Let T be an undirected graph. Then any 2 of the following properties $\Rightarrow$ the 3rd

1). T is connected

2) T has no cycles

2) T has $n-1$ edges

Just proved $1+2 \Rightarrow 3$

Prove: $\boxed{\begin{array}{c} 1+3 \Rightarrow 2 \\ 2+3 \Rightarrow 1 \end{array}}$