Sep 26

PROPOSITION: Let T be a BFS tree for $G=(V,E)$

If $(u,w) \in E$ s.t. $u \in L_i$, $w \in L_j$

$\Rightarrow |i-j| \leq 1$

Pf (idea) By contradiction

For the sake of contradiction

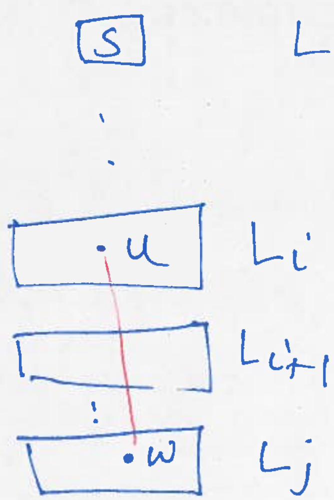
$|i-j| > 1$ (but $j \geq i$)

$\Rightarrow$ ~~j~~ $j > i+1$

or $j \geq i+2$

W.l.o.g. / WLOG (Without loss of generality) assume $i \leq j$

(otherwise switch the role of u & w)



Consider the iteration where BFS is constructing L_{i+1}.

$\rightarrow$ we know $u \in L_i$

$w \notin L_0, \ldots, L_i$

BUT $(u,w) \in E$

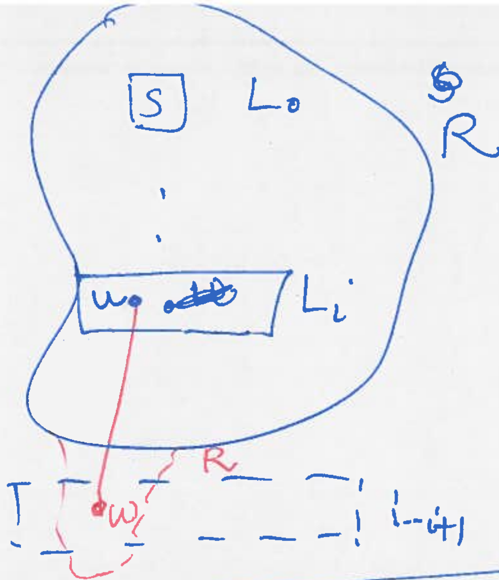
$\Rightarrow$ by definition of BFS

w should be included in L_{i+1}

$\Rightarrow$ contradiction ∎

Explore(s)

0. ~~R~~ $R = \{s\}$
1. While $\exists (u,w) \in E$ s.t. $u \in R$, $w \notin R$
   Add w to R
2. Output $R^* = R$.



<u>Def:</u> The set of all vertices in G that are connected to s is called the connected component of s (Denote it by $CC(s)$)

<u>THM 1:</u> $R^* = CC(s)$

<u>General trick:</u> $A = B \iff A \subseteq B$ and $B \subseteq A$

<u>Lemma 1:</u> $R^* \subseteq CC(s)$ ← Ex. Prove by induction. At any stage of the algo, $R \subseteq CC(s)$

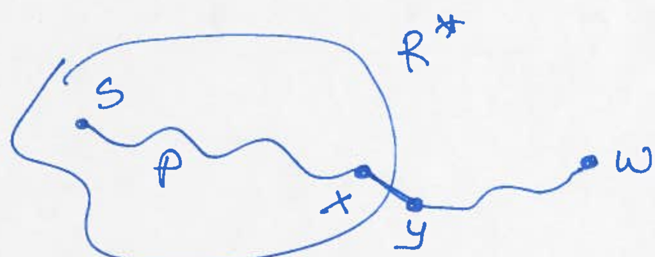
<u>Lemma 2:</u> $CC(s) \subseteq R^*$

Lemmas 1+2 $\Rightarrow$ THM 1

<u>Pf (idea) of Lemma 2:</u> By contradiction $(CC(s) \not\subseteq R^*)$

$\Rightarrow \exists \underbrace{w \in CC(s)}$ but $w \notin R^*$

$\Rightarrow \exists$ $s-w$ path P



Since ~~then~~ P starts off inside R^* (at s) & ends up outside of R^* (at w) $\exists\, x \in R^*$, $y \notin R^*$ and $(x,y) \in E$

(Note: $y = w$ is allowed)

by algo defn, the algo has not yet terminated.

$\Rightarrow$ contradiction ~~as~~ existence of $R^* \Rightarrow$ algo has terminated. ∎