Oct 10 $S^* \to$ output of greedy algo
$\mathcal{O} \to$ an optimal schedule

THM 2: $|S^*| = |\mathcal{O}|$

Notation: $S^* = \{i_1, \dots, i_k\}$ $\quad \mathcal{O} = \{j_1, \dots, j_m\}$

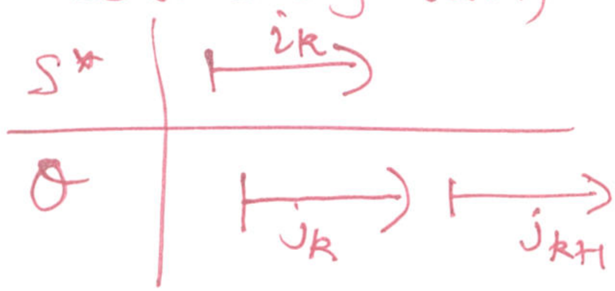
$f(i_1) \leq f(i_2) \dots \leq f(i_k)$ $\quad f(j_1) \leq \dots \leq f(j_m)$

THM 2': $k = m$ $\quad$ [Claim: $k \leq m$ as $\mathcal{O}$ is an optimal soln]

LEMMA 1: "Greedy stays ahead" $\quad \forall\ 1 \leq l \leq k,\ f(i_l) \leq f(\cancel{i_l}\ j_l)$

Pf (idea) of THM 2': By contradiction.

Assume (by Claim) $k < m \iff m \geq k+1$



By Lemma 1, $f(j_k) \geq f(i_k)$

Also $\exists\ j_{k+1} \in \mathcal{O}$

(Ex.) Claim 2: j_{k+1} does not conflict with $i_1, \dots, i_k$

$\Rightarrow$ After i_k is added to S, $j_{k+1} \in R$

$\Rightarrow R \neq \{\} \Rightarrow$ Greedy algo could not have terminated

$\Rightarrow$ contradiction.

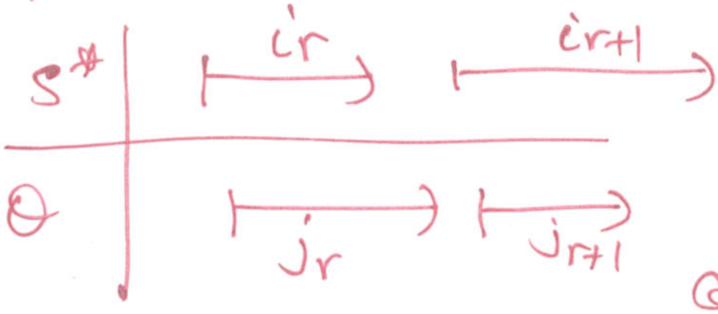
Pf (idea) of Lemma 1: Prove by induction on l.

Base case: $f(i_1) \leq f(j_1)$ {as the 1st job added by Greedy to S has the smallest finish time}.

Assume: $f(i_l) \leq f(j_l)\ \forall l \leq r$

Inductive step: want to show $f(i_{r+1}) \le f(j_{r+1})$

for the sake contradiction assume $f(i_{r+1}) > f(j_{r+1})$



By I.H. $f(i_r) \le f(j_r)$

Consider the time after i_r is added to S

~~Clea~~ Ex: j_{r+1} does not conflict with $i_1, \ldots, i_r$

$\Rightarrow$ Greedy algo considered $j_{r+1} \in R$, but chose $i_{r+1} \in R$

$\Rightarrow f(i_{r+1}) \le f(j_{r+1}) \Rightarrow$ contradiction ∎

Greedy Algo

$f(1) \le f(2) \le \cdots \le f(n)$

0. $R = [n]$
1. $S = \{\}$

$\}\ O(n)$

2. While $R \ne \{\}$ $\leftarrow \le n$ times

$O(n) \rightarrow$ (2.1) Pick $i \in R$ with the smallest index

$O(1) \rightarrow$ (2.2) Add i to S

$O(n) \rightarrow$ (2.3) Remove all $j \in R$ that conflicts with i

3. Return $S^* = S$ $\}\ O(n)$

Overall: $O(n^2)$

Next class: $O(n \log n)$