Oct 28

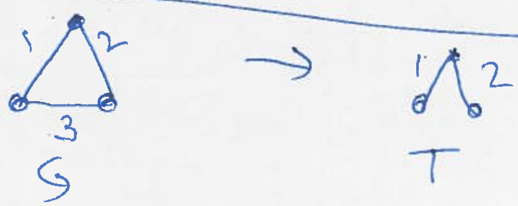
Minimum Spanning Tree (MST) problem

Input: $G = (V,E)$, $c_e \geq 0$ [mostly for convenience] $\forall e \in E$

(G is connected)

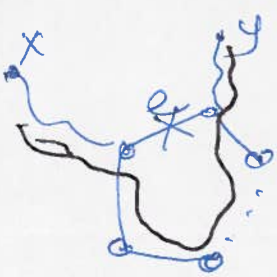
Output: (i) $E' \subseteq E$ s.t. $T = (V, E')$ is connected. ↖ [T is subgraph of G]

(ii) min $c(T) = \sum_{e \in E'} c_e$



Prop: Let $c_e > 0$ $\forall e \in E$. Then the optimal solution T is a tree.

Pf (idea): for contradiction assume that T is not a tree $\Longrightarrow$ (as T is connected) T has a cycle.



Let $e \in C$

↳ Delete e from T to get T'

Claim 1: T' is still connected

Claim 2: $c(T') < c(T)$

$c(T') = c(T) - c_e < c(T)$ ← as $c_e > 0$

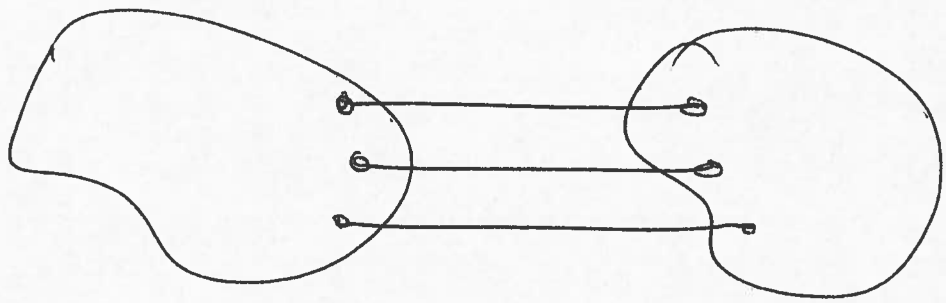
contradicts the fact that T is optimal $\Longleftarrow$

Brute force algo: Go over all possible subsets E' of E & check if (V, E') is connected ↖ 2^m of those.

keep track of one w/ min cost

CUT PROPERTY LEMMA

Assume all c_e's are distinct



$S \neq \{\}$ $\qquad$ $V \setminus S \neq \{\}$

$(S, V \setminus S)$ $\qquad$ $(V \neq S \neq \{\})$

a CUT

If e is the crossing edge w/ min cost

$\Rightarrow e$ is in ALL MSTs for G.