~~Aug~~ Sep 2 — Stable matching / Marriage (NOT feminist)

n men $M = \{m_1, \ldots, m_n\}$

n women $W = \{w_1, \ldots, w_n\}$

$\boxed{n=2}$ Example

$M = \{BP, BBT\}$

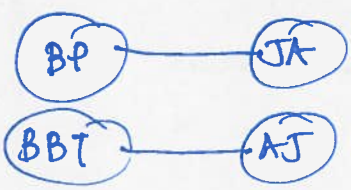
$W = \{JA, AJ\}$

Def (Matching) A subset of pairs $S \subseteq M \times W$ (def $\{(m,w) \mid m \in M, w \in W\}$) is a matching if

(i) $\forall$ (for all) $m \in M$, $\exists$ (there exists) at most one $w \in W$ s.t. (such that) $(m,w) \in S$

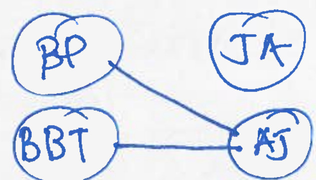
(ii) $\forall w \in W$, $\exists$ ——— $m \in M$ s.t. $(m,w) \in S$

Def (perfect) matching ↑ replace at most 1 by exactly 1.



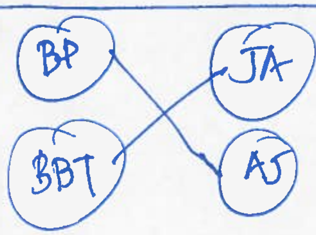
✓P ✓M

$S = \{(BP, JA), (BBT, AJ)\}$



✗P ✗M

✗P ✓M



✓P ✓M

Aside: (Ex) There are $n!$ perfect matching

Preference list: $\forall m \in M$, L_m: total ranking of all women in W

$\forall w \in W$, L_w: ——— men in M

↑ total of $2n$ preference list

Example:

L_{BP}: AJ > ~~BBT~~ JA

L_{BBT}: AJ > JA

L_{JA}: BP > BBT

L_{AJ}: BP > BBT

total size of all pref list $= 2n^2$

(Def) Stable matching: A stable matching S is s.t.

(1) S is a perfect matching

(2) It has no instability

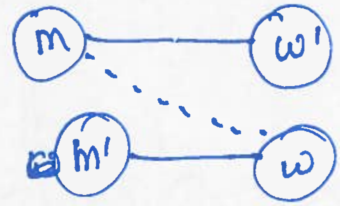
(given a perfect matching S & $2n$ preferences $\forall m \in M$ $\forall w \in W$)

A pair (m, w) is an instability if

(1) $(m, w) \notin S$

(2) In L_m: $w > w'$

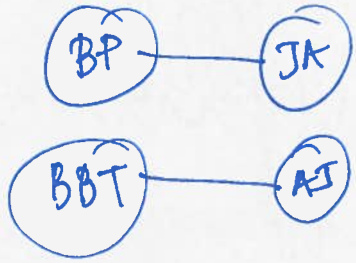
(3) In L_w: $m > m'$



New pref list

L'_{BP}: $AJ > JA$

L'_{BBT}: $JA > AJ$



(BP, AJ) is an instability

Problem: $|M| = |W| = n$

Input: M, W & $2n$ pref lists: L_m $\forall m \in M$, L_w $\forall w \in W$

Output: A stable matching (if it $\exists$)