Subset sum I/p: n number $w_1, \dots, w_n$ ← $O(\log n)$ bits
a bound W } $\log W$ bits n ~~regis~~ register/mem locati

$\Rightarrow$ # register/ locatn $= \Theta\left(n + \frac{\log W}{\log n}\right)$

run time: $O(nW)$ W is not $\text{poly}\left(\frac{\log W}{\log n}\right)$

input size $\Theta(n) \Leftarrow$ for say $W = 2^n$

but $\rightarrow O(nW) = O(n2^n)$

not poly(n)

Similar example: given integer $n \geq 1$ check if it is prime: Algo: check if i divides n for $2 \leq i \leq \lfloor\sqrt{n}\rfloor$

runtm $= \Theta(\sqrt{n})$ but input size $= \Theta(\log n)$ bits.

$\sqrt{n} = \sqrt{2^{\log n}} = 2^{\frac{\log n}{2}} = (\sqrt{2})^{\log n}$

Q 3 on Sample final: $G = (V, E)$ $\forall e \in E, 0 \leq p_e \leq 1$ $\forall p_e = \left(\frac{1}{2}\right)^{w_e}$

$s, t \in V$

$\Pr(P) = \prod_{i=1}^{\ell} p_{e_i}$ largest $\Pr(P)$ $\forall$ s-t paths P

$P \downarrow e_1, \dots, e_\ell$

Claim 1: Let $w(P) = \sum_{e \in P} w_e$

~~The path P than max Pr(P) also min w(P)~~

In fact $\Pr(P) = \left(\frac{1}{2}\right)^{w(P)}$

Claim 2: That path P that max $\Pr(P)$ also min $w(P)$

Claim 1+2 $\Rightarrow$ Algo ① Compute P than min $w(P)$ by Dijkstra

② Output $\left(\frac{1}{2}\right)^{w(P)}$

Pf (idea) Claim 1: By Defn

$$Pr(P) = \prod_{i=1}^{\ell} \left(\frac{1}{2}\right)^{we_i} \overset{(*)}{=} \left(\frac{1}{2}\right)^{\sum_{i=1}^{\ell} we_i} = \left(\frac{1}{2}\right)^{w(P)}$$

Argument for $(*)$ for $\ell = 2$

$$\left(\frac{1}{2}\right)^{we_1} \cdot \left(\frac{1}{2}\right)^{we_2} = \left(\frac{1}{2}\right)^{we_1 + we_2}$$

$\log_2(a \cdot b) = \log_2 a + \log_2 b$

take logs $\log_2$

$$\Leftrightarrow \log\left(\frac{1}{2}\right)^{we_1} + \log\left(\frac{1}{2}\right)^{we_2} = \log_2\left(\frac{1}{2}\right)^{we_1 + we_2}$$

$$\Leftrightarrow \log_2 2^{-we_1} + \log_2 2^{-we_2} = \log_2 2^{-(we_1 + we_2)}$$

$$\Leftrightarrow -we_1 - we_2 = -(we_1 + we_2) \checkmark$$

Same pf works for $a^b \cdot a^c = a^{b+c}$

Pf (idea) of Claim 2: ~~if~~ P $\max_P Pr(P)$

~~then~~ $\Leftrightarrow$ P maximizes $\log_2 Pr(P)$

Claim 1 $\Leftrightarrow$ P max $\log_2 \left(\frac{1}{2}\right)^{w(P)}$

$\Leftrightarrow$ P max $-w(P)$

$\Leftrightarrow$ P min $w(P)$.

In gen replace p_e by $\left(\frac{1}{2}\right)^{\underbrace{\log(1/p_e)}_{=we}}$