

Sep 7

Gale-Shapley algo

Structure:

- ① Initially all $2n$ men & women are free
- ② In a loop: a free woman propose to a man
- ③ You have n matched pairs (diff from book)

Initial state: All n men & n women are free

① Let w be a free woman w

Q: Who should w propose to?

A: w should propose to her top choice m : $L_w: m > \dots$

② w proposes to m

Q: What should m do?

Case 2.1: m accepts w 's proposal & they get matched

Problem: What if m gets a "better" proposal later?

Case 2.2: m rejects w 's proposal

Problem: What if m does not get a better proposal later?

Case 2.3 (m, w) get engaged.

Note: At any stage all men or women are either free or engaged.

In general: ① If all men & women are engaged
 \Rightarrow Algo terminates
 \Rightarrow engaged pairs are the matched pairs.

① ~~if~~ otherwise \exists a free woman w

Q: Who should w propose to?

A: Let m be w 's top man that she has NOT proposed to yet.

② w proposes to m

Q: What ~~should~~ should m do?

Case 2.1: m is free $\Rightarrow (m, w)$ get engaged

Case 2.2: (m, w') are engaged

Case 2.2.1: $w > w'$ in L_m
 (m, w) get engaged
 w' is free

Case 2.2.2: $w' > w$ in L_m

\notin no change $[(m, w')$ are still engaged
 w is still free]