

Sep 9

THEOREM: For every input (or instance)

the GS algo outputs

$\left\{ \begin{array}{l} M, W, 2n \\ \text{pref lists} \\ n = |M| = |W| \end{array} \right\}$

a stable matching.

COROLLARY: Every instance of the stable matching problem has a stable matching.

LEMMA 1: On every input, the GS algo terminates in $\leq n^2$ iterations.

LEMMA 2: The output of GS algo (S) is a perfect matching.

LEMMA 3: S has no instability.

LEMMA 1 + 2 + 3 \Rightarrow THEOREM

Pf of LEMMA 1

Proof idea: By a progress measure \rightarrow

Argue in every iteration a new proposal is made

But # proposals \leq # pairs $(m, w) = |M \times W| = n^2$ (*)

Pf details: At the end of iteration t ,

$P(t)$ = total # of proposals made so far.

We'll argue

(1) $P(1) = 1$ \leftarrow by algo definition (because a free woman proposes to her top man)

(2) $\forall t \quad P(t+1) = P(t) + 1$ \leftarrow by ~~algo~~ algo defn (itr $t+1$ happens since w had not proposed to all men).

(3) $\forall t \quad P(t) \leq n^2$ (by (*))

claim: (1) + (2) + (3) \Rightarrow Lemma 1.