Sep 12 LEMMA 2: S is a perfect matching.

Obs 0: S is a matching

Obs 1: Once a man gets engaged, he keeps on getting engaged to better & better women.

Obs 2: If w proposes to m AFTER m' then $m' > m$ in L_w

LEMMA 4: If at the end of an iteration, w is free $\Rightarrow$ w has NOT proposed to all men.

Pf of LEMMA 2:

(Pf idea) Proof by contradiction (Use Obs 0, Lemma 4, algo defn)

(Pf detail) For the sake of contradiction, assume S is NOT a perfect matching.

$\Rightarrow$ $\exists$ a free woman w (follows from Obs 0, algo defn)

$\Rightarrow$ (by Lemma 4) $\exists$ a man m that w has NOT proposed to. ——— (*)

Since algo has output S, it has terminated

$\Rightarrow$ (by conds in loop) there is no free woman w has has not proposed to all men.

$\Rightarrow$ contradicts (*) ∎

Pigeon-hole principle: If $\leq n-1$ pigeons are put into n holes $\Rightarrow \exists \geq 1$ empty hole.

Pf of Lemma 4

(Pf idea) Proof by contradiction (use pigeonhole principle + Obs 1)

(Pf details) For sake of contradiction, assume
$\exists$ a free woman w who has proposed to ~~all~~ ALL men.

$\Rightarrow$ all men are engaged (follows from Obs 1 + Algo statement)
(*)

Since w is free $\Rightarrow \leq n-1$ engaged women

$\Longrightarrow$ $\leq n-1$ engaged men (≥ 1 free man)

pigeon-hole principle
hole :: men
pigeon :: women
assign :: engaged.

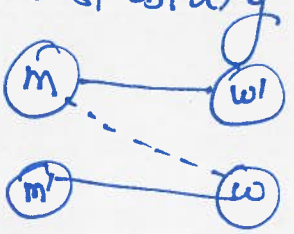
$\Rightarrow$ contradicts (*) ∎

LEMMA 3: S has no instability.

(Pf idea) Pf by contradiction (Use Obs 1, Obs 2, Lemma 2) w/ case analysis.

(Pf details) Assume S has an instability (m, w)

AND (i) $w > w'$ in L_m
(ii) $m > m'$ in L_w



Case 1: w never proposed to m.

Since (m', w) are engaged $\Rightarrow$ w proposed to m' (but never to m)

$\Rightarrow$ $m' > m$ in L_m $\Rightarrow$ contradicts (i)
by Obs 2