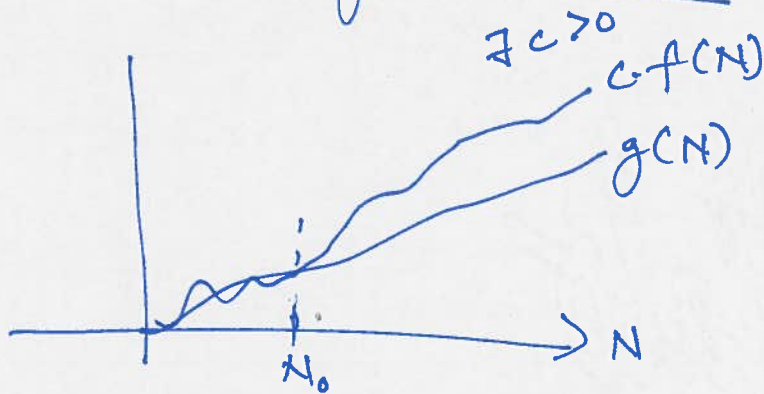


Sep 14

Big-O notation



$g(N)$ is $O(f(N))$

$$10N+1 \stackrel{?}{=} O(\sqrt{N})^x$$

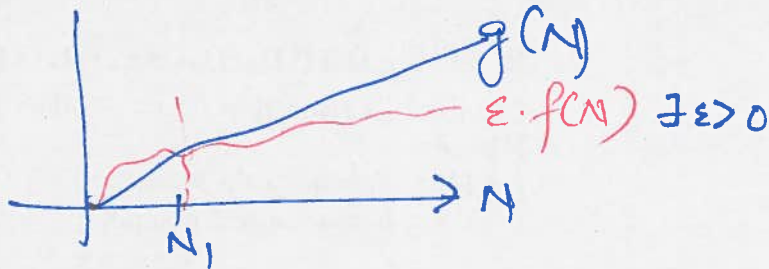
$$O(N) \checkmark$$

$$O(N^2) \checkmark$$

$$O(N!) \checkmark$$

Big-Omega notation

$g(N)$ is $\Omega(f(N))$



$$10N+1 \stackrel{?}{=} \Omega(\sqrt{N})$$

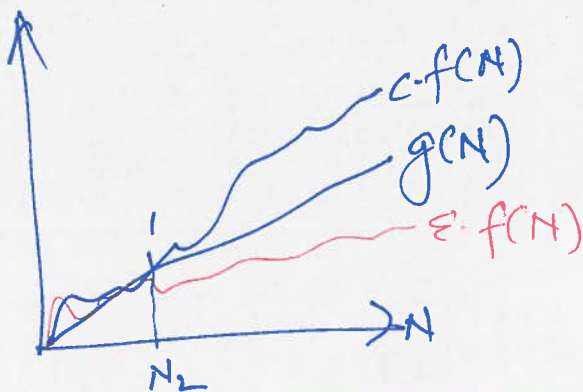
$$\Omega(N)$$

$$\Omega(N^2)$$

$$\Omega(N!)$$

Big-Theta notation

$g(N)$ is $\Theta(f(N))$ if $g(N)$ is $O(f(N))$ AND $g(N)$ is $\Omega(f(N))$



$$10N+1 \stackrel{?}{=} \Theta(\sqrt{N})^x$$

$$\Theta(N) \checkmark$$

$$\Theta(N^2)^x$$

$$\Theta(N!)^x$$

Properties of O: (Also valid for Ω & Θ)

① Transitive: $g(N)$ is $O(f(N))$ and $f(N)$ is $O(h(N)) \Rightarrow g(N)$ is $O(h(N))$

$$(10N+1 \text{ is } O(N)) \ \& \ N \text{ is } O(N^2) \Rightarrow 10N+1 \text{ is } O(N^2)$$

② Additive: $f(N)$ is $O(K(N))$ AND $g(N)$ is $O(K(N))$
 $\Rightarrow f(N) + g(N)$ is $O(K(N))$

$10N$ is $O(N)$ & 1 is $O(N)$ \Rightarrow ~~$10N+1$~~ is $O(N)$

③ Multiplicative: If $f(N)$ is $O(h_1(N))$ and $g(N)$ is $O(h_2(N))$
 $\Rightarrow f(N) \cdot g(N)$ is $O(h_1(N) h_2(N))$

$$\begin{array}{ccc} N \cdot \overset{10}{\cancel{10}} & = & O(N) \\ \uparrow \quad \uparrow & & \\ O(N) & & O(1) \end{array}$$