Sep 16 We'll usually use asymptotic notation to talk about runtimes of algos but defns we have seen are INDEPENDENT of what $f(n)$ "means"

Eg $f(N)$ = # times Akash says no to me on his Nth day of existence.

$T_A(N) = \underline{\text{max}}$ # of steps the algo A takes on $\underline{\text{any}}$ input of size N.

Talk about $T(N)$ is $O(f(N))$ or $\Omega(g(N))$

Example: $N = n+1 = \theta(n)$ ← return any i s.t. $a_i == v$, -1 if none $\exists$.

SEARCH $(a_0, \ldots, a_{n-1}; v)$

$\leq n$ times → for $(i=0; i<n, i{+}{+})$ ← T_0 = # times the loop runs $\quad T_0 \leq n$ ($\forall$ inputs)

$O(n)$ $O(1)$ { if $(a_i == v)$ return i } T_1 : # steps for check $\quad T_1 \leq O(1)$ ($\forall$ inputs)

$O(1)$ → return -1 } T_2 : # steps to return -1 $\quad T_2 \leq O(1)$ ($\forall$ inputs)

overall = $O(n)$

$$T(N) \leq T_0 \cdot T_1 + T_2 \leq \underbrace{n \cdot O(1)}_{\leq O(n)} + \underbrace{O(1)}_{O(n)} \leq O(n) + O(n) \leq O(n)$$

multiplicativity $\quad$ Additivity

Claim: $T(N)$ is $O(n)$

$T(N) = \max\{$ # steps taking by A on 1st i/p of size N,
________ 2nd ________
________ 3rd ________
... $\}$

$\Rightarrow$ $\forall$ ~~inputs~~ i, A on ith input takes $\leq T(N)$ steps.

We argued, if $\forall i$, A on ith input takes $\leq u$ steps $O(n)$

$\Rightarrow T(N) \leq u \, (O(n))$

for SEARCH.

Claim: $T_{SEARCH}(N) \geq \Omega(n)$ $\left(\Rightarrow T(N) \text{ is } \Theta(n) = \Theta(N\right.$

$T(N) \geq L \iff \exists\, i$ s.t. A on ith input $\geq L$ steps.

<u>Proof idea/strategy</u>: Exhibit <u>one</u> input of size $n+1 = N$ s.t. SEARCH takes $\Omega(n)$ steps on that input.

<u>Pf details</u>: Consider $a_i = i$ $0 < i < n$

$v = n$

runtime of algo $\geq T_0 \cdot T_1 \geq n \cdot 1 \geq \Omega(n)$ ∎

<u>Implementing GS algo</u>

<u>Outline</u>: $\boxed{\text{Initialization}}$ $\leftarrow T_0$

while (..) $\leftarrow T_1$ is # itr $\leq n^2$

$\boxed{\text{Body of loop}}$ $\} \; T_2$

$\boxed{\text{Output } S}$ $\} \; T_3$

$T(N) \leq T_0 + T_1 \cdot T_2 + T_3$

show this!

If we can argue $T_0, T_3 \leq O(n^2)$, $\boxed{T_2 \leq O(1)}$

$\Rightarrow T(N) \leq O(n^2) + n^2 \cdot O(1) + O(n^2)$

$\leq O(n^2)$.

Notation change: Assum $M = [n] \overset{def}{=} \{1, ., n\}$ $W = [n]$

woman $i \to i^{th}$ woman

man $i \to i^{th}$ man