Recitation 1 (9/6-9/9)

Overview:
Introductions, OH times, HW tips, why this course is the bomb, etc.
Stable Matching Background
Proof by Induction
Proof by Counterexample
Proof by Contradiction

Stable Matching Review:

The Problem
Input: set of n men $M = \{m_1, m_2, m_3, \dots m_n\}$
set of n women $W = \{w_1, w_2, w_3, \dots w_n\}$
L_m = total ranking of all women
L_w = total ranking of all men
Output: A stable matching

Stable Matching: A perfect matching that has no instabilities

Perfect Matching: $\forall$ m $\in$ M, m is assigned exactly 1 woman AND
$\forall$ w $\in$ W, m is assigned exactly 1 woman

Instability: Given two pairs (m, w) and (m', w'), where m prefers w' to w and w' prefers m to m' the pair (m, w') is an instability

Proof by Induction:
HW0 Q2
Prove that the total number of perfect matchings when you have n men and n women is n!.
(Recall n!=n×(n−1)×(n−2)×⋯×2×1n!=n×(n−1)×(n−2)×⋯×2×1.)

Perfect matching ≠ Stable matching
In a perfect matching we do not care about preferences, just that every individual has a match

e.g.: $\{w_1, w_2, w_3\}$ Perfect matching: $\{(m_1, w_1), (m_2, w_2), (m_3, w_3)\}$
$\{m_1, m_2, m_3\}$

m_1 can be with 3 women, m_2 with 2, m_3 with the last 3*2*1 = 3!

***Note: when thinking about HW problems, its almost ALWAYS super helpful to make e.g.s**

Induction Proofs have 3 steps

1. Base Case (n = 1)
2. Inductive Hypothesis (n = k-1)
3. Induct! (Prove for n = k)

*Note: these values are not concrete, they can change depending on problem statement

Base Case: Prove for n=1. M={m_1} W={w_1}. The only perfect matching is (m_1, w_1) so there is 1 perfect matching. 1 = 1! so there are 1! = n! perfect matchings.

Inductive Hypothesis: Assume that for n = k-1 there are (k-1)! perfect matchings.

Inductive Step: Prove that for n = k there are n! = k! perfect matchings. M = {m_1, m_2, … m_k} and W = {w_1, w_2, … w_k}. Consider m_1. There are k possible choices for m_1. After the initial matching, regardless of choice there are (k-1) men and (k-1) women left. From the inductive hypothesis, we know that there are (k-1)! perfect matchings when n = (k-1). Therefore the total number of matchings is k*(k-1)! = k*(k-1)*(k-2)*(k-3)*...1 = k! = n!.

Goal in proof by induction: Use the inductive hypothesis to help you solve the larger problem at hand.

Proof by Contradiction

(Not to be confused with proof by Counterexample)

Prove x.

Assume !x to prove x.

Aside (proof by counterexample):

Counterexample**:** when you are able to prove a point by giving an example that makes the statement false

e.g. Prove that… is or is not true.

If you are proving that a statement is not true, you can often use a counterexample.

This is NOT always the case, but most common case of counterexamples

e.g.

There is no possible stable matching where everyone matches with his/her first choice.

Prove this statement to be true or false.

M = {m_1, m_2, m_3}

W = {w_1, w_2, w_3}

m_1: $w_1 > w_2 > w_3$	w_1: $m_1 > m_2 > m_3$
m_2: $w_2 > w_3 > w_1$	w_2: $m_2 > m_3 > m_1$
m_3: $w_3 > w_1 > w_2$	w_3: $m_3 > m_1 > m_2$

Example:

Question:
Assume that the following are true:

• Every blockbuster movie has a hero.
• Jake Sully and Neytiri are dating.
• The highest grossing movie ever is a blockbuster.
• A hero in a movie never dies.
• The movie Avatar has made the most money ever.
• The heroine always dates the hero.
• Neytiri is the heroine of Avatar.

<u>Prove by contradiction</u> that Jake Sully is alive at the end of the movie Avatar. Please clearly state any assumptions that you needed to make in the proof.

Proof:
Assume the opposite of what you are trying to prove.
Assume that Jake Sully is NOT alive at the end of the movie Avatar.

Assuming that Jake Sully is dead at the end of the movie Avatar, then we must assume that Jake Sully is not the hero of Avatar since the hero is a movie never dies.

In order to prove that Avatar has a hero at all, we need to show that show that Avatar is a blockbuster movie. Since the highest grossing movie ever is a blockbuster, and the movie Avatar has made the most money ever, we can conclude that it must have a hero, but that hero can not be Jake Sully from our previous assumption.

Now we know that Neytiri is the heroine of Avatar and that the herione always dates the hero. Since Jake Sully is not the hero of Avatar, we can assume that Neytiri is not dating Jake Sully. This directly contradicts the statement from the problem's assumptions "Jake Sully and Neytiri are dating." This results in a contradiction and our initial assumption must be false, so we know that Jake Sully must be alive at the end of Avatar.