Recitation 2 (9/11-9/15)

Reminders:

Remember to put collaborators AND sources on your pdfs even if there are none
Can only collaborate with 2 other people & must be for entire homework
Proof Idea & Proof Details (clearly labeled)

Finding all stable matchings

Remember: Gale-Shapley is only helpful for finding a stable matching (2 if different depending on if women propose and if men propose) but not all of them

Example:

m_1: $w_1 > w_2 > w_3$	w_1: $m_2 > m_3 > m_1$
m_2: $w_2 > w_3 > w_1$	w_2: $m_3 > m_1 > m_2$
m_3: $w_3 > w_1 > w_2$	w_3: $m_1 > m_2 > m_3$

What are the stable matches?

[$(m_1,w_1),(m_2,w_2),(m_3,w_3)$]
[$(m_2,w_1),(m_3,w_2),(m_1,w_3)$]
[$(m_1,w_2),(m_2,w_3),(m_3,w_1)$] → Not Gale-Shapley

What do we need to do to find all stable matchings?
~Back to the definition

A stable matching is: a perfect matching with no instabilities

Brute force algorithm:

1. Find all perfect matchings
2. Check that the perfect matching has no instabilities

Step 1: Find all perfect matchings

All perfect matchings can be found based off of the permutation of n. (We proved in last week homework that the total number of perfect matching is n!).

Permutation of {m_1, m_2, m_3}

6 possible permutations:

m_1 m_2 m_3
m_1 m_3 m_2
m_2 m_1 m_3
m_2 m_3 m_1
m_3 m_1 m_2
m_3 m_2 m_1

But what about the women?
By keeping the women in numerical order, you are creating a different matching every time. Hence why the the total number of perfect matchings is just n!.

Step 2: Check for instability

An instability: Any time there is a pairing (m,w) and (m',w') such that m prefers w' to w and w' prefers m to m'.
In other words, there is at least one pair of couples where the husband from Couple 1 and the wife from Couple 2 (or vice versa) prefer each other to their current spouses.
(cred to Piazza posters)

<u>Your job</u>: Figure out how to turn this information into a somewhat efficient algorithm.

Example Proof: Chapter 1 Ex. 1

Decide whether the following is true or false. If true, prove it. If it is false, give a counterexample.

In *every* instance of the Stable Matching Problem, there is a stable matching containing a pair (m,w) such that m is ranked first on the preference list of w and w is ranked first on the preference list of m.

$m_1 = \{w_2 > w_1\}$ $w_1 = \{m_1 > m_2\}$
$m_2 = \{w_1 > w_2\}$ $w_2 = \{m_2 > m_1\}$

False.
Proof Idea: In order to prove this statement, it is sufficient to provide a counterexample to the statement above. We will show that there is an instance of the stable matching problem that does not contain the pair (m,w) where m is ranked first on the preference list of w and w is ranked first on the preference list of m.

Proof Details: The example below is an instance of the stable matching problem since there are n men and n women where every m in M and every w in W has a preference list of n women and men, respectively.

$m_1 = \{w_2 > w_1\}$ $w_1 = \{m_1 > m_2\}$
$m_2 = \{w_1 > w_2\}$ $w_2 = \{m_2 > m_1\}$

There are only 2 possible matchings and only 2 stable matchings for this instance of the stable matching problem $[(m_1,w_2), (m_2,w_1)]$ and $[(m_1,w_1), (m_2,w_2)]$. A pair such that both partners rank their partner first does not exist for this problem.

Pigeon Hole Principle

Consider any assignment of n +1 pigeons to n holes. Then there exists at least one hole with at least 2 pigeons in it.

Example:
There are 500 students in a school. By pigeonhole principle, there must be at least 2 students that share a birthday.

For the proof, more examples, and variations of the principle see the support pages:
http://www.cse.buffalo.edu/faculty/atri/courses/331/fall14/handouts/proof-primer.pdf