

# Lecture 28

CSE 331

Nov 6, 2017

# Mini project video due next Mon

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## Video submission now open on Autolab

Sorry, forgot to do this earlier: you can now submit your video (note still PDF with the link in it) on Autolab.

**YOU WILL NEED TO FORM YOUR GROUP ON AUTOLAB AGAIN BEFORE SUBMITTING.**

See the mini project page for the details:

<http://www-student.cse.buffalo.edu/~atri/cse331/fall17/mini-project/index.html>

#pin

mini\_project

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# Anonymous feedback

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## 331 Feedback

If you have the time, please do fill in this feedback form:

[https://docs.google.com/forms/d/e/1FAIpQLSeZldd6oBwjXeH3YBR6f6cxCVgOph1ialwtj47LGBLT-aSpOw/viewform?usp=sf\\_link](https://docs.google.com/forms/d/e/1FAIpQLSeZldd6oBwjXeH3YBR6f6cxCVgOph1ialwtj47LGBLT-aSpOw/viewform?usp=sf_link)

Filling in the form is optional and is anonymous. But your feedback would be very helpful. If you have limited time, I would encourage you to at least fill in the questions on the initiatives that are new to this Fall.

In a few weeks I will summarize some of the feedback and try and respond to the common comments/questions.

#pin

feedback

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# Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

“Patch up” the solutions to the sub-problems for the final solution

# Improvements on a smaller scale

Greedy algorithms: exponential  $\rightarrow$  poly time

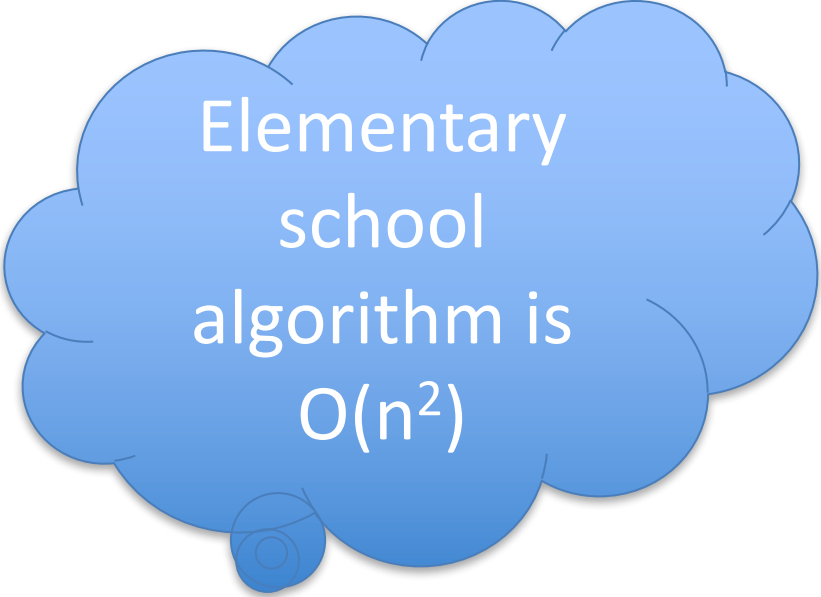
(Typical) Divide and Conquer:  $O(n^2)$   $\rightarrow$  asymptotically smaller running time

# Multiplying two numbers

Given two numbers  $a$  and  $b$  in binary

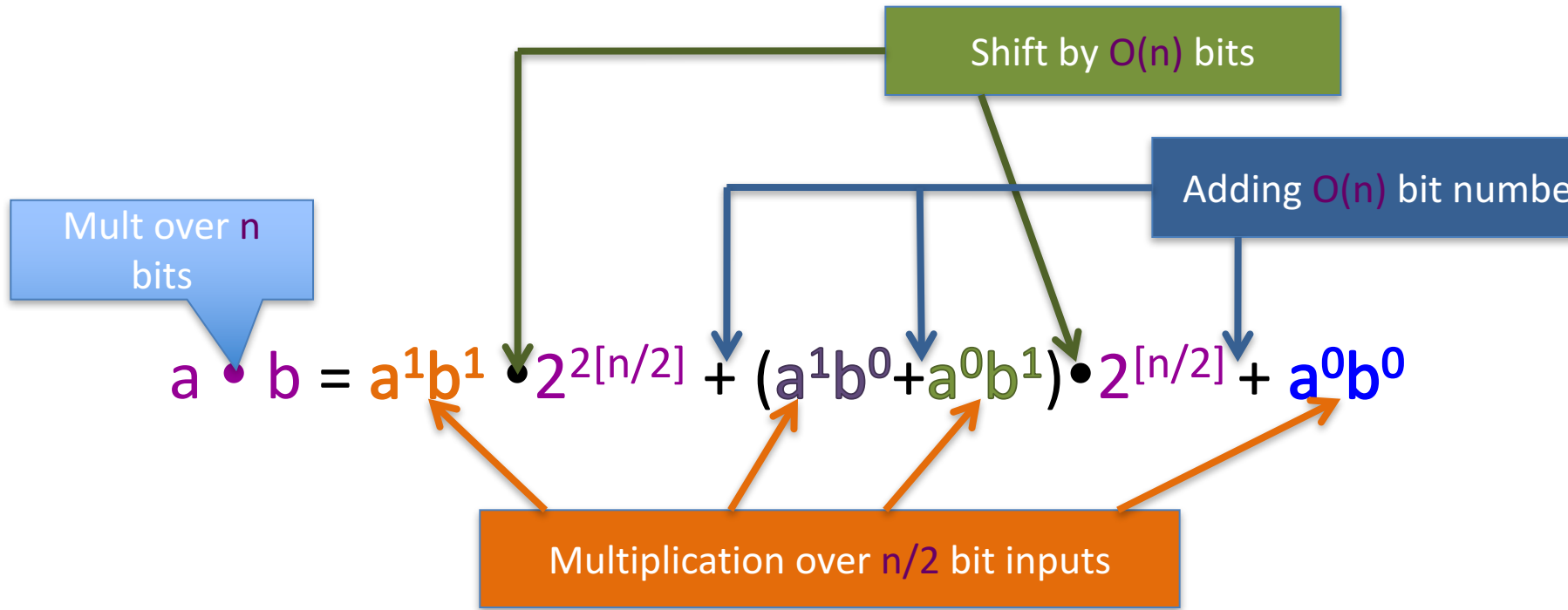
$$a = (a_{n-1}, \dots, a_0) \text{ and } b = (b_{n-1}, \dots, b_0)$$

Compute  $c = a \times b$



Elementary  
school  
algorithm is  
 $O(n^2)$

# The current algorithm scheme



$$T(n) \leq 4T(n/2) + cn \dots$$

$$T(1) \leq c$$

$T(n)$  is  $O(n^2)$

# The key identity

$$a^1b^0 + a^0b^1 = (a^1 + a^0)(b^1 + b^0) - a^1b^1 - a^0b^0$$



# The final algorithm

Input:  $a = (a_{n-1}, \dots, a_0)$  and  $b = (b_{n-1}, \dots, b_0)$

**Mult** ( $a, b$ )

If  $n = 1$  return  $a_0 b_0$

$a^1 = a_{n-1}, \dots, a_{\lfloor n/2 \rfloor}$  and  $a^0 = a_{\lfloor n/2 \rfloor - 1}, \dots, a_0$

Compute  $b^1$  and  $b^0$  from  $b$

$x = a^1 + a^0$  and  $y = b^1 + b^0$

Let  $p = \text{Mult}(x, y)$ ,  $D = \text{Mult}(a^1, b^1)$ ,  $E = \text{Mult}(a^0, b^0)$

$F = p - D - E$

return  $D \cdot 2^{2\lfloor n/2 \rfloor} + F \cdot 2^{\lfloor n/2 \rfloor} + E$

$$T(1) \leq c$$

$$T(n) \leq 3T(n/2) + cn$$

$O(n^{\log 3}) = O(n^{1.59})$   
run time

All **green** operations  
are  $O(n)$  time

$$a \cdot b = a^1 b^1 \cdot 2^{2\lfloor n/2 \rfloor} + ((a^1 + a^0)(b^1 + b^0) - a^1 b^1 - a^0 b^0) \cdot 2^{\lfloor n/2 \rfloor} + a^0 b^0$$