Lecture 4

CSE 331

Sep 6, 2017

Read the syllabus CAREFULLY!

No graded material will be handed back till you submit a signed form!

CSE 331 **Introduction to Algorithm Analysis and Design** **Fall 2017**

University at Buffalo
Department of Computer Science & Engineering
CSE 331 — Introduction to Algorithm Analysis and Design

- Make sure you fill in form **with a pen.**
- After you have filled in the form, scan it and upload it to Autolab.

I, _____ (PRINT name), acknowledge that I have read and understood the syllabus (and the homework policy document) for this course, CSE 331 *Introduction to Algorithm Analysis and Design.*

Sign-up for mini projects

Deadline: Monday, Sep 25, 11:59pm

Email me your group (=3) composition

Separate Proof idea/proof details

</> Note

Notice how the solution below is divided into proof idea and proof details part. **THIS IS IMPORTANT: IF YOU DO NOT PRESENT A PROOF IDEA, YOU WILL NOT GET ANY CREDIT EVEN IF YOUR PROOF DETAILS ARE CORRECT.**

Proof Idea

As the hint suggests there are two ways of solving this problem. (I'm presenting both the solutions but of course you only need to present one.)

We begin with the approach of reducing the given problem to a problem you have seen earlier. Build the following complete binary tree: every internal node in the tree represents a "parent" RapidGrower while its two children are the two RapidGrowers it divides itself into. After s seconds this tree will have height s and the number of RapidGrowers in the container after s seconds is the number of leaf nodes these complete binary tree has, which we know is 2^s. Hence, the claim is correct.

The proof by induction might be somewhat simpler for this problem if you are not comfortable with reduction. In this case let $R(s)$ be the number of RapidGrowers after s seconds. Then we use induction to prove that $R(s) = 2^s$ while using the fact that $2 \cdot 2^s = 2^{s+1}$.

Proof Details

We first present the reduction based proof. Consider the complete binary tree with height s and call it $T(s)$. Further, note that one can construct $T(s+1)$ from $T(s)$ by attaching two children nodes to all the leaves in $T(s)$. Notice that the newly added children are the leaves of $T(s+1)$. Now assign the root of $T(0)$ as the original RapidGrower in the container. Further, for any internal node in $T(s)$ ($s \geq 0$), assign its two children to the two RapidGrowers it divides itself into. Then note that there is a one to one correspondence between the RapidGrowers after s seconds and the leaves of $T(s)$. Then we use the well-known fact (cite your 191/250 book here with the exact place where one can find this fact): $T(s)$ has 2^s leaves, which means that the number of RapidGrowers in the container after s seconds is 2^s, which means that the claim is correct.

TA office hours finalized

note

stop following 75 views

Actions

TA office hours

The timings for TA office hours have been fixed. The syllabus has been updated with this information (and it is also re-produced at the end of the post).

Some remarks:

- For now all the TA office hours are scheduled in the TA areas near **Davis 302**. Some of the office hours will be scheduled in other places: we will update this information once this is done.
- Some of the office hours have a language associated with them. In such office hours Q1 questions related to the specific programming language will take precedence. In all other homeworks, questions on Q2 and Q3 will take precedence over Q1. (Of course this is the rubric used by the TA to prioritize: if you are the only person in the office hour then they will help you with whatever question(s) you might have.)
- Some of the office hours this Wed (Sep 6) could be canceled: be on the lookout for those announcements.
- In a few days, the office hours should also be on the 331 Google calendar (which you can see in the 331 homepage).

TA	OHs
Anand	Mon 11am-12pm (Python), Tue 5-6pm
Aishani	Tue 3-4pm (Python), Wed 5-6pm
Adhish	Mon 3-4pm, Th 11am-12pm
Sravanika	Mon 11am-12pm, Wed 4-5pm, Wed 5-6pm
Katie	Th, 4-5pm, Th 5-6pm
Dhruv	Tue 5-6pm (Java), Wed 3-4pm
Kevin	Tue 2-3pm (C++), Th 2-3pm

Makeup recitations

TODAY, 4-5pm in Davis 338A

Tomorrow, 5-6pm in Davis 113A

On matchings

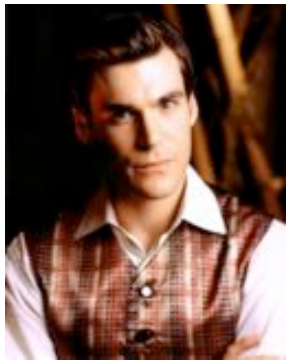
Mal

Inara

Wash

Zoe

Simon

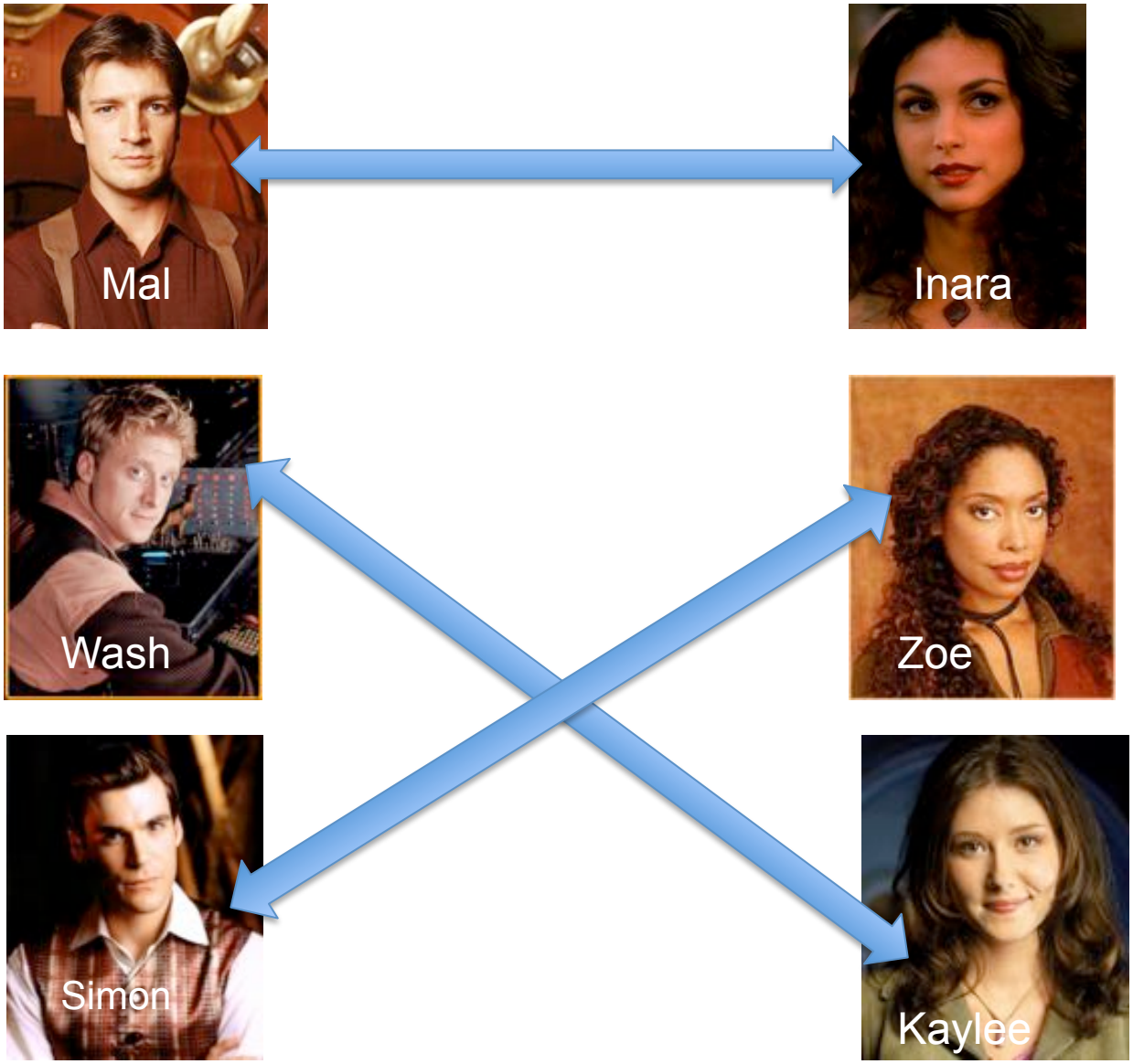


Kaylee

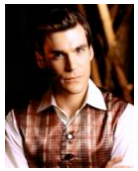
A valid matching

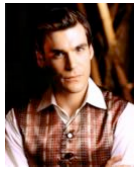
Not a matching

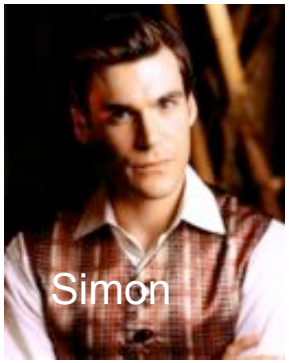
Perfect Matching

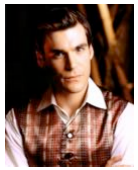


Preferences

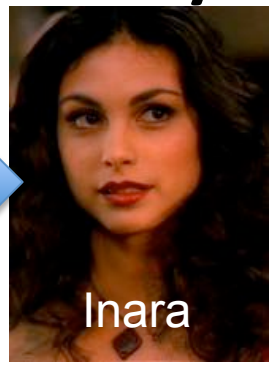


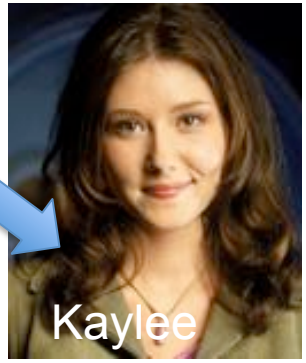






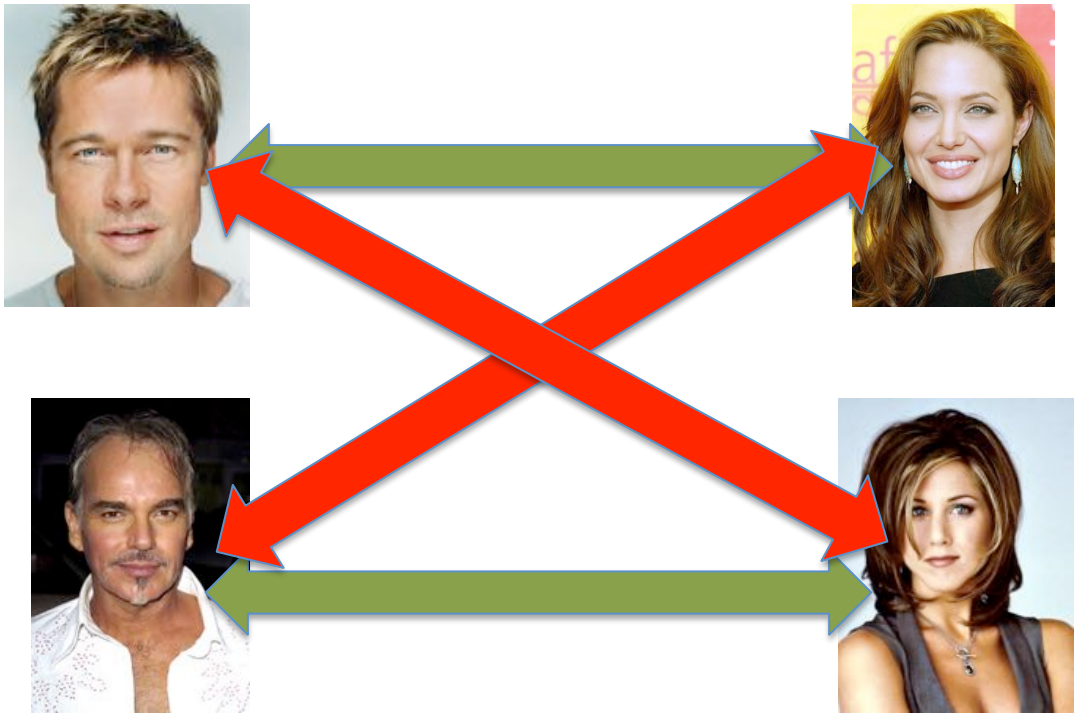
Instability



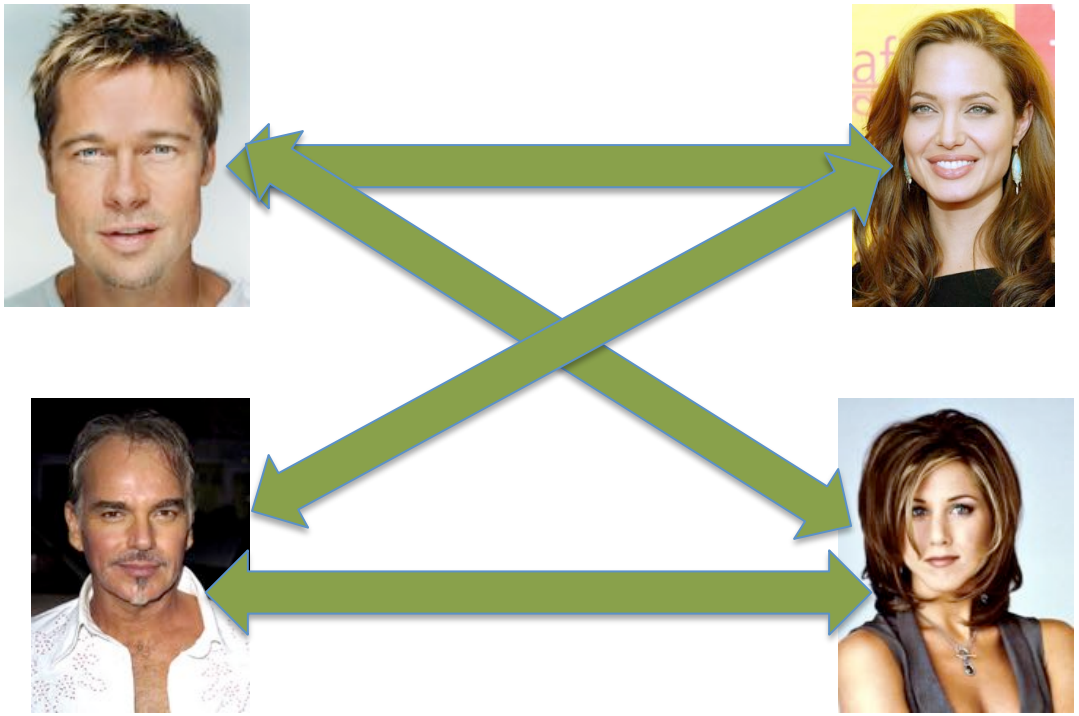


A stable marriage

Even though BBT and JA are not very happy



Two stable marriages



Stable Marriage problem

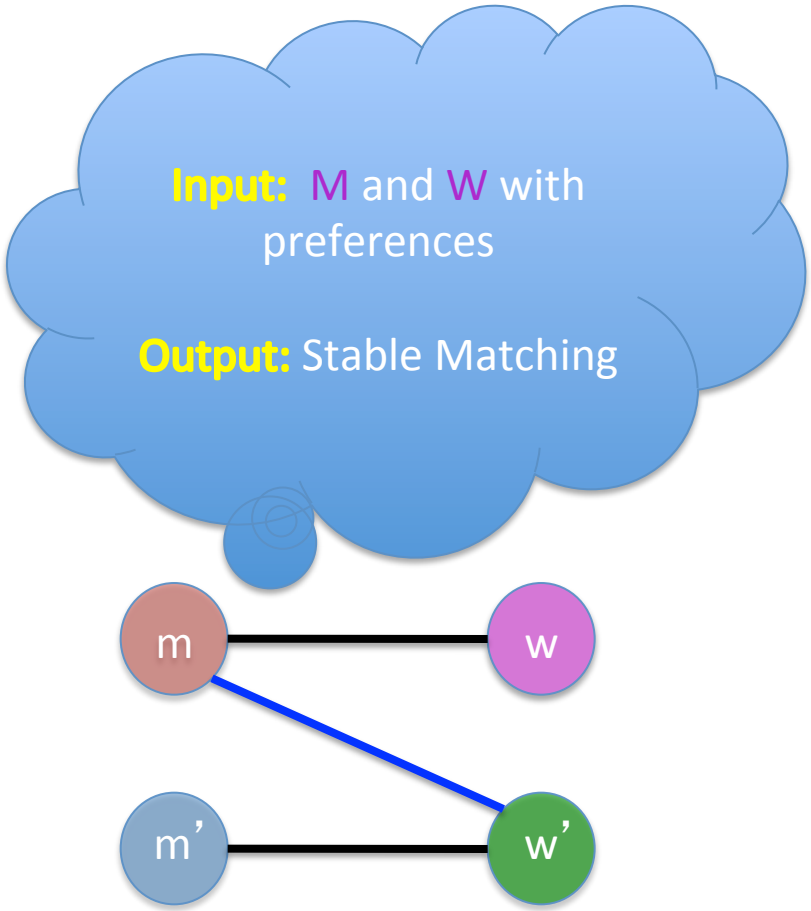
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instablity



Stable matching = perfect matching+ no instablity

Questions/Comments?

Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

Today's lecture

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

The naïve algorithm

Incremental algorithm to produce all n! prefect matchings?

Go through all possible perfect matchings S

If S is a stable matching

then Stop

Else move to the next perfect matching

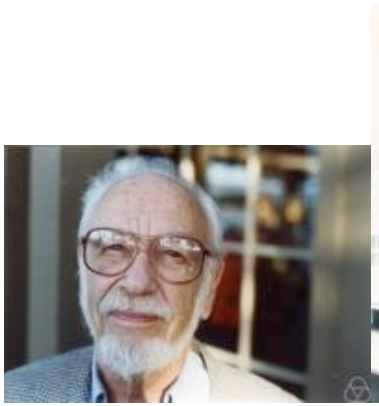
Gale-Shapley Algorithm

David Gale

Lloyd Shapley

$O(n^3)$ algorithm

Moral of the story…



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Questions/Comments?

Gale-Shapley Algorithm

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

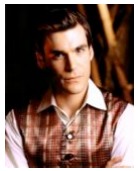
w remains free

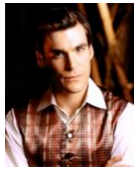
Else

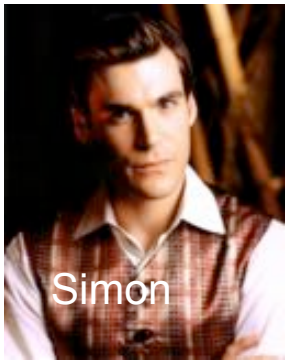
(m,w) get engaged and w' is free

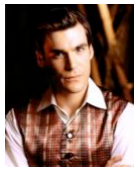
Output the engaged pairs as the final output

Preferences









GS algorithm: Firefly Edition

Observation 1

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Once a man gets engaged, he remains engaged (to "better" women)

Output the engaged pairs as the final output

Observation 2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

If w proposes to m after m', then she prefers m' to m

Output the engaged pairs as the final output

Questions/Comments?

Why bother proving correctness?

Consider a variant where any free man **or** free woman can propose

Is this variant any different? Can you prove it?

GS’ does not output a stable marriage