Lecture 6

CSE 331

Sep 11, 2017

Mini project group due in 2 weeks

CSE 331 Mini project choices

Fall 2017

Please check the table below before submitting your mini project pitch to make sure your case study is not being used by another group. Case studies are assigned on a first come first serve basis.

Group	Societal Aspect	Case Study
Yong Yang Chen, Shally Guo and Levy Shi		
Stephen James, Angus Lam and Daniel Stamer		
Gary Dos Santos, Hui Gao and Darasy Reth		
Jimmy Huang, Connor Reynolds and William Stewart		
Harshita Girase, Heeba Karlapper and Doris Kwan		
Tammy Chang, Shubham Singh and Alex Stewart		

Gale-Shapley Algorithm

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

  w proposes to m

  If m is free

    (m,w) get engaged

  Else (m,w') are engaged

    If m prefers w' to w

      w remains free

    Else

      (m,w) get engaged and w' is free

Output the engaged pairs S as the final output

Observation 1

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Once a man gets engaged, he remains engaged (to "better" women)

Output the engaged pairs S as the final output

Observation 2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w’) are engaged

If m prefers w’ to w

w remains free

Else

(m,w) get engaged and w’ is free

If w proposes to m after m’, then she prefers m’ to m

Output the engaged pairs S as the final output

Proof via "progress"

Intially all men and women are free

While there exists a free woman who can propose

> Let w be such a woman and m be the best man she has not proposed to
>
> w proposes to m
>
> If m is free
>
> (m,w) get engaged
>
> Else (m,w') are engaged
>
> If m prefers w' to w
>
> w remains free
>
> Else
>
> (m,w) get engaged and w' is free

Output the engaged pairs as the final output

End of iteration t, define progress P(t) s.t.:

1. $1 \leq P(t) \leq n^2$
2. $P(t+1) = P(t)+1$

Today's lecture

GS algorithms always outputs a stable marriage

The Lemmas

Lemma 1: The GS algorithm has at most n^2 iterations

Lemma 2: S is a perfect matching

Lemma 3: S has no instability

Questions/Comments?

Extensions

Fairness of the GS algorithm

Different executions of the GS algorithm

Main Steps in Algorithm Design

Problem Statement

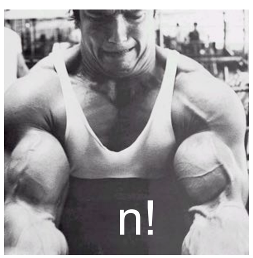
NRMP
National Resident Matching Program

↓

Problem Definition

↓

Algorithm



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"Implementation"

↓

Analysis

Correctness Analysis

Definition of Efficiency

An algorithm is efficient if, when implemented, it runs quickly on real instances

Implemented where?

Platform independent definition

What are real instances?

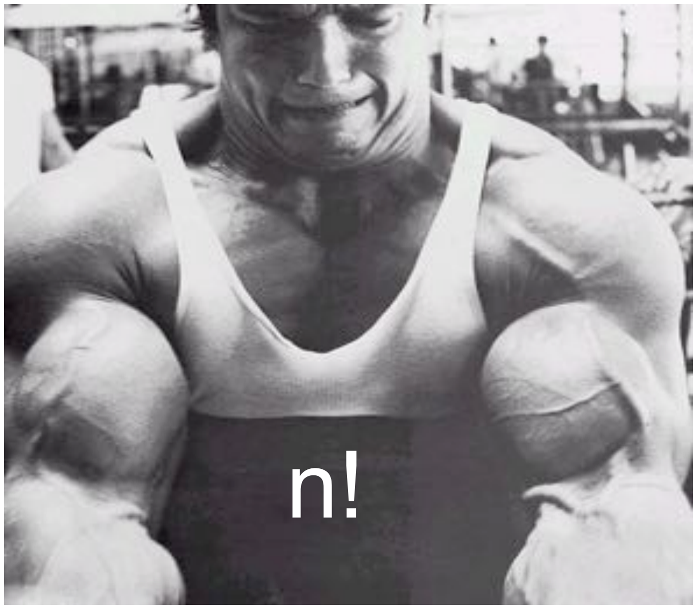
Worst-case Inputs

$N = 2n^2$ for SMP

Efficient in terms of what?

Input size N

Definition-II



Analytically better than brute force

How much better? By a factor of 2?

Definition-III

Should scale with input size

If N increases by a constant factor, so should the measure

Polynomial running time

At most $c \cdot N^d$ steps ($c>0$, $d>0$ absolute constants)

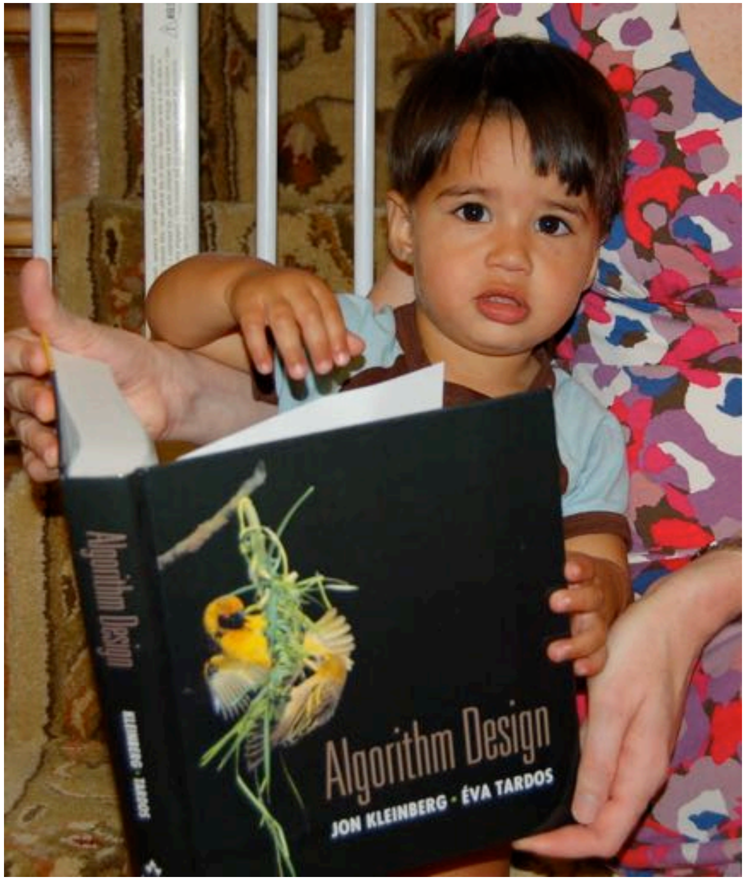
Step: “primitive computational step”

More on polynomial time

Problem centric tractability

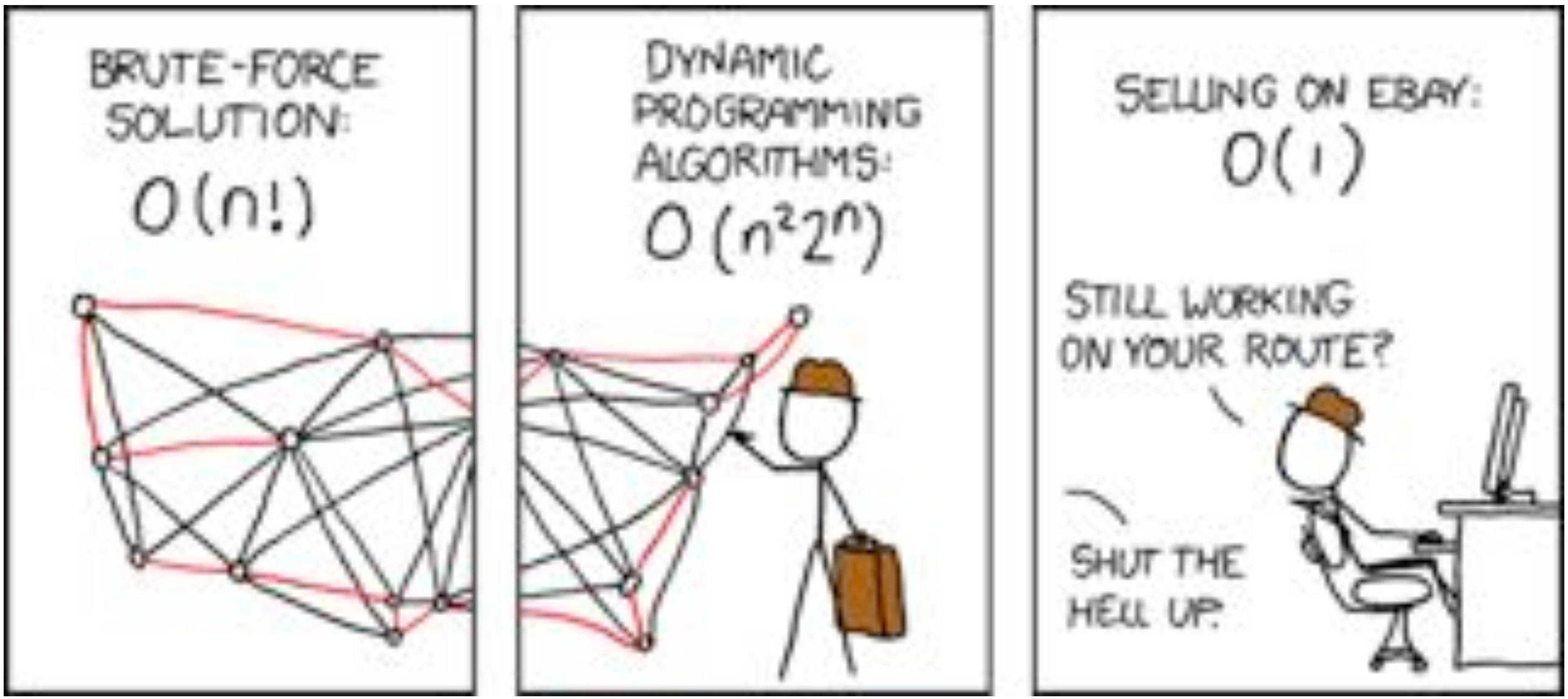
Can talk about problems that are not efficient!

Reading Assignments



Sections 1.2, 2.1, 2.2 and 2.4 in [KT]

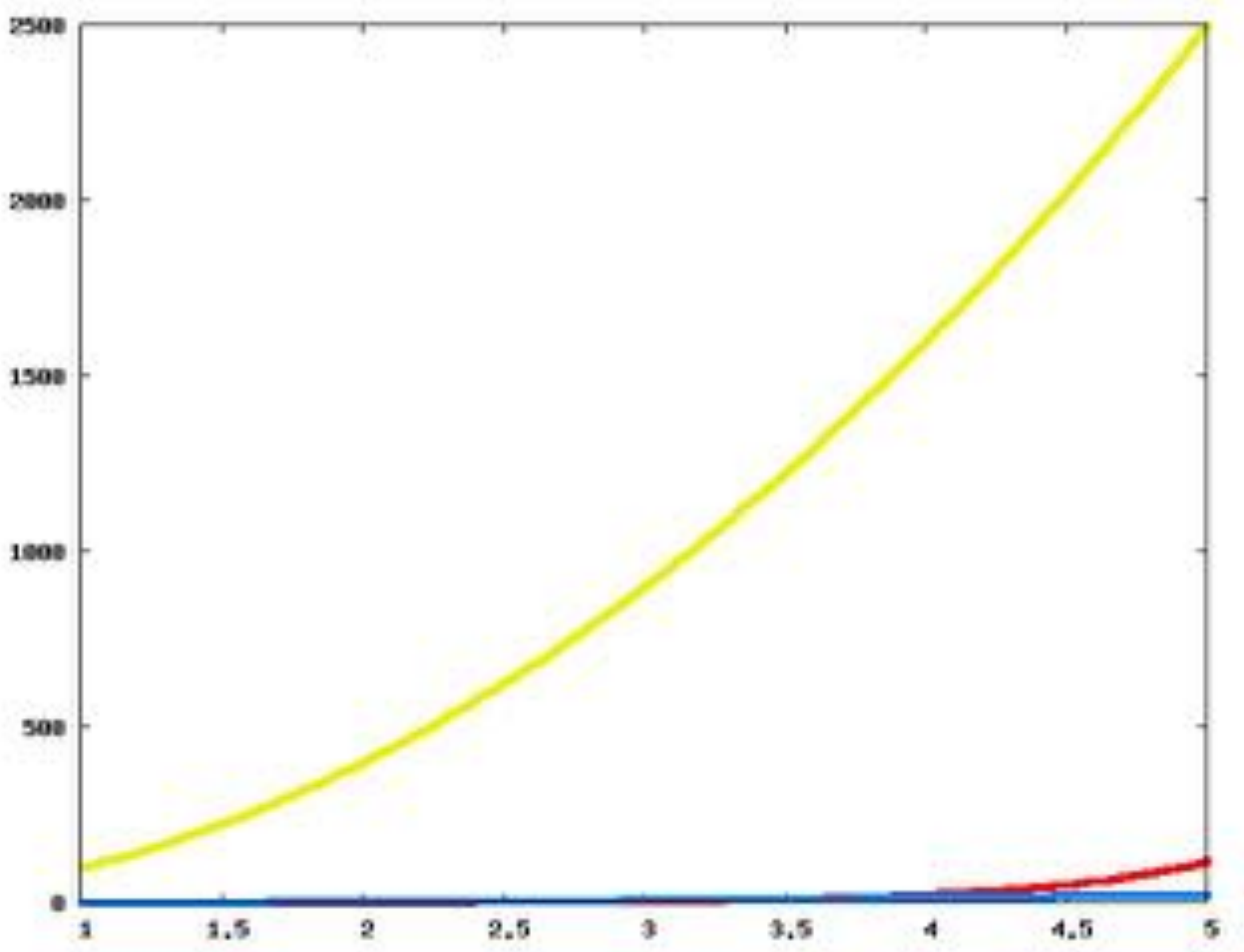
Asymptotic Analysis



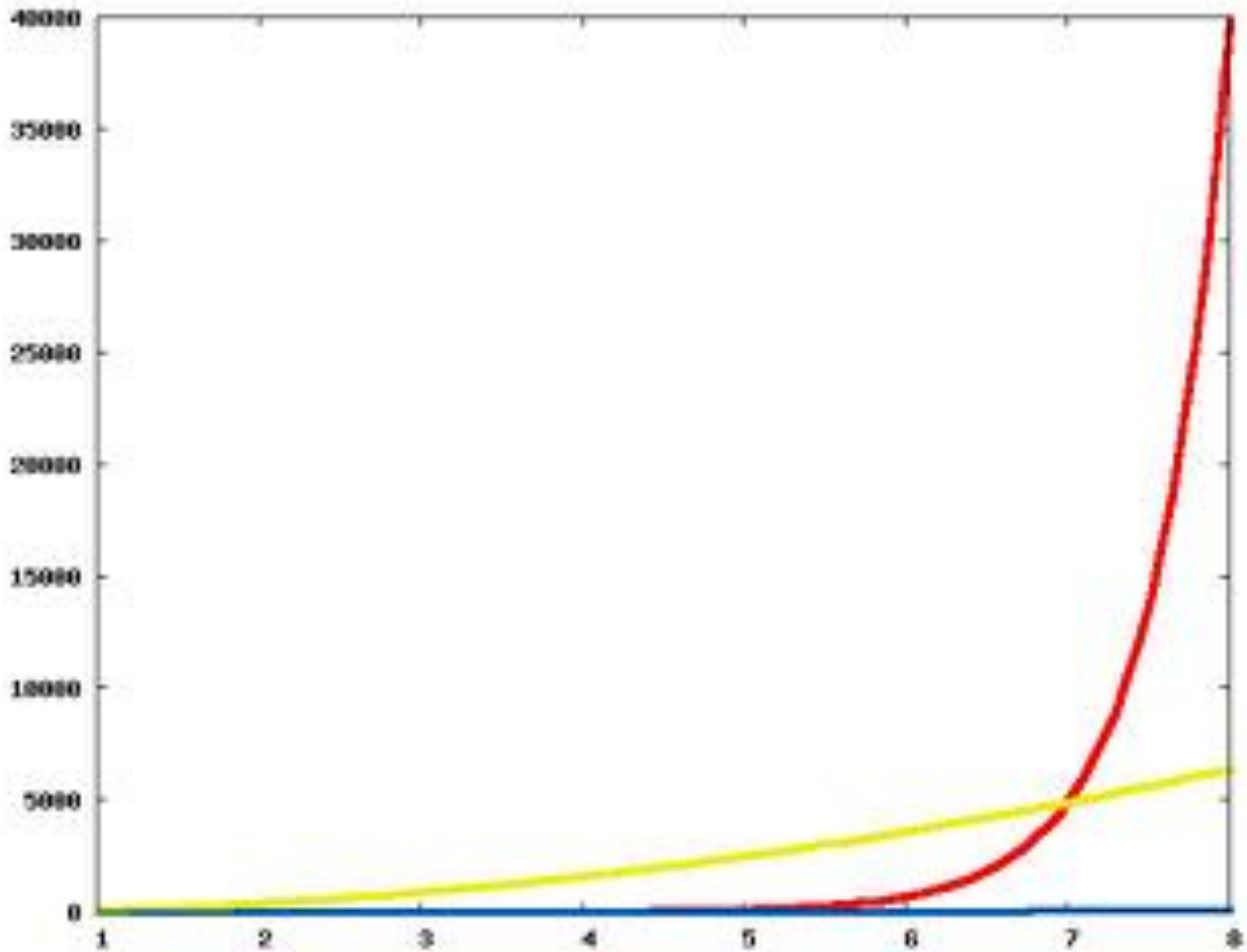
Travelling Salesman Problem

(http://xkcd.com/399/)

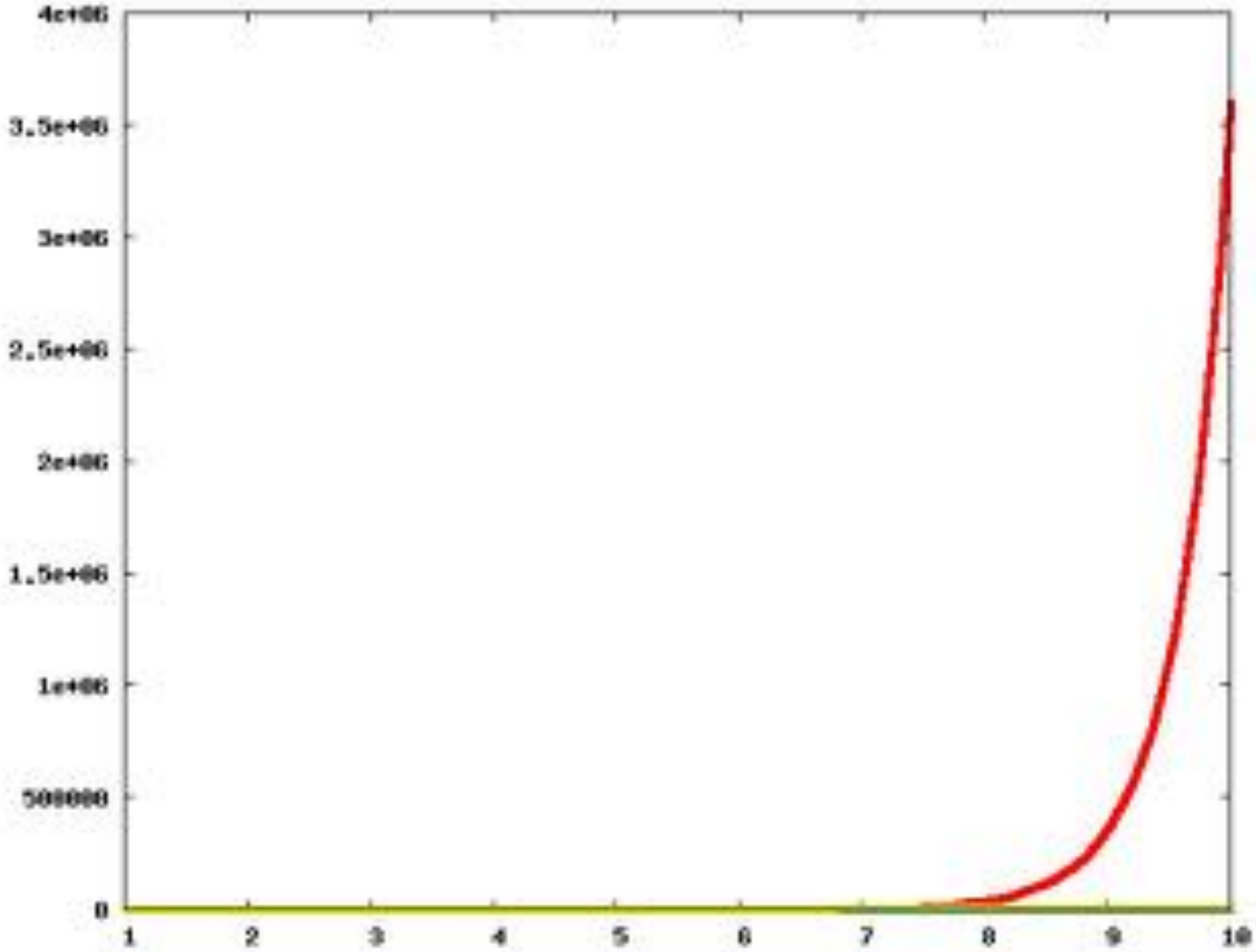
Which one is better?



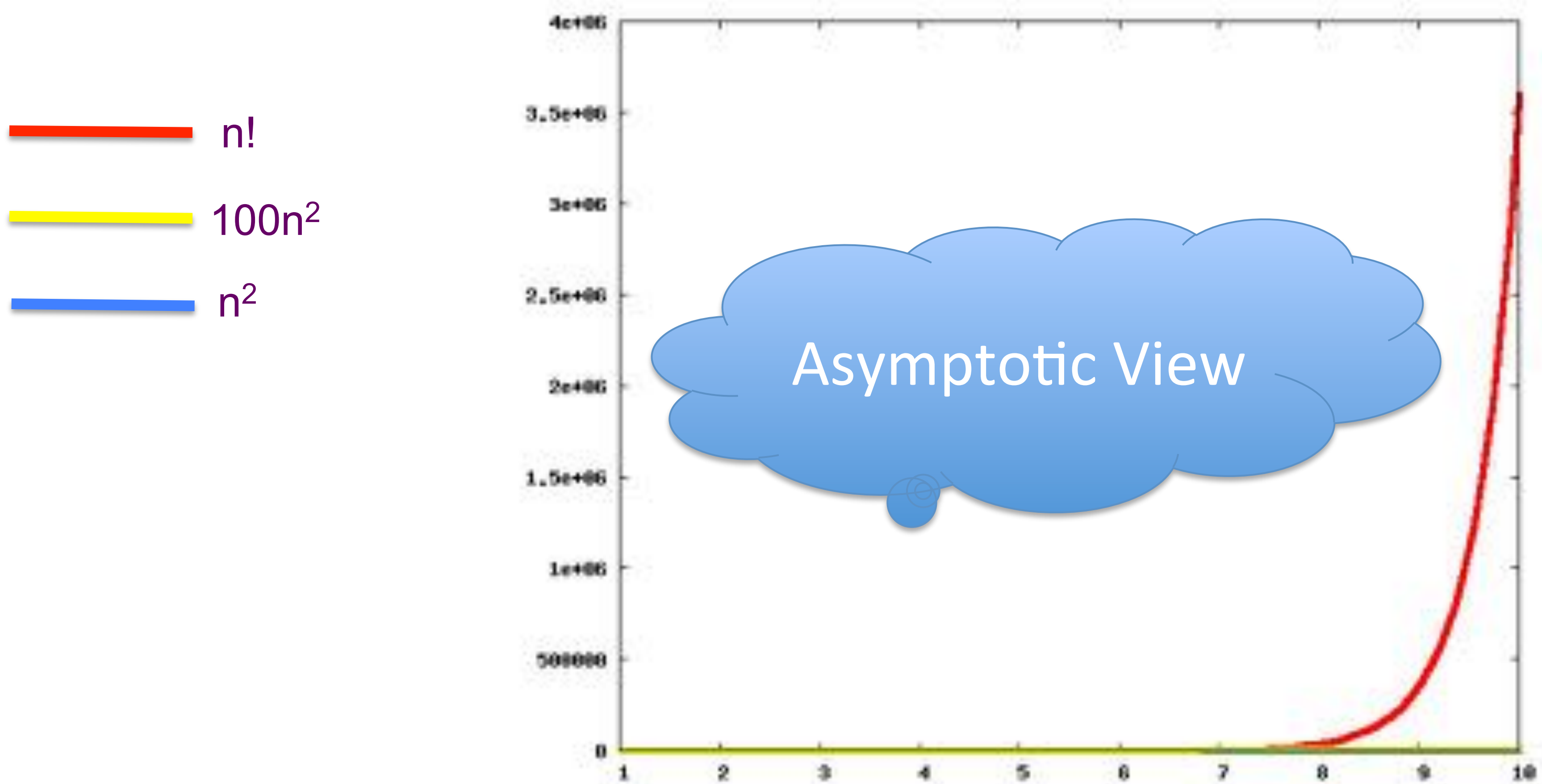
Now?



And now?



The actual run times



Asymptotic Notation

$\leq$ is O with glasses

$\geq$ is Ω with glasses

= is Θ with glasses