$\boxed{\text{Sep 25}}$

PROPOSITION: Let T be a BFS tree for $G=(V,E)$

If $(u,w) \in E$ s.t. $u \in L_i$, $w \in L_j$

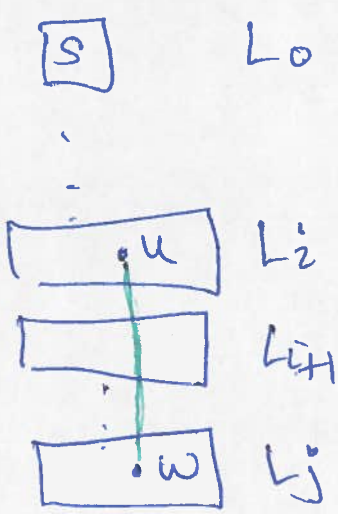
$\Rightarrow |i-j| \le 1$ $\left(\overset{\Longleftrightarrow}{\text{equivalent}} \; i \in \{j-1, j, j+1\} \right)$

Pf (idea) {Proof details in book} Pf by contradiction.

$\left.\begin{array}{l}\text{WLOG} \\ \text{w.l.o.g.}\end{array}\right\}$ Without loss of generality.

WLOG assume $i \le j$ (If not, do argument after switching i & j).

for contradiction assume $|i-j| > 1 \Rightarrow j > i+1$ or $j \ge i+2$



Consider situation when BFS is creating L_{i+1}.

$\rightarrow u \in L_i$; $w \notin L_0, \dots, L_i$

$\rightarrow (u,w) \in E$

$\Rightarrow w$ satisfies condition to be added in L_{i+1}

$\Rightarrow$ contradicts $w \in L_j$ $j > i+1$

Explore (s)

0. $R = \{s\}$
1. While $\exists$ $v \notin R$, $u \in R$, $(u,v) \in E$
   Add v to R
2. Output $R^* = R$

Def: The set of all vertices connected to s (in G) is the connected component of s. Denote it by $CC(s)$

THEOREM: $R^* = CC(s)$

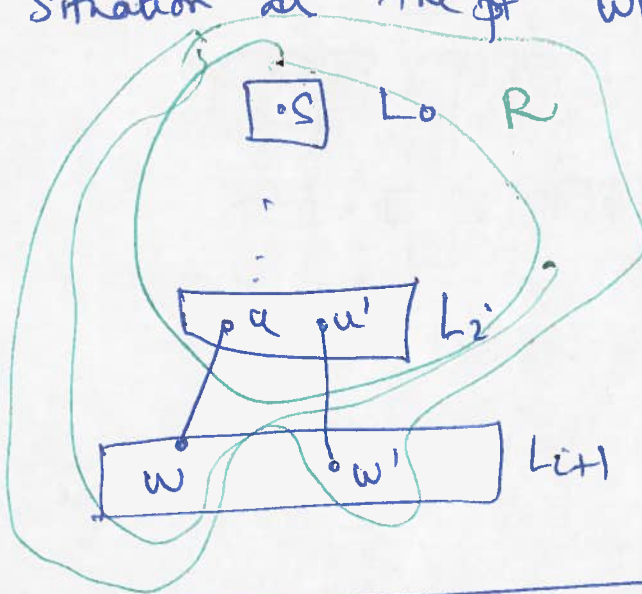
Note: If $G=(V,E)$ is connected $\Rightarrow \forall s \in V$, $CC(s) = V$.

General trick: $A = B$ (sets)

$\Longleftrightarrow A \subseteq B$ and $B \subseteq A$

Note: BFS is a special case of Explore.

Consider the situation at the pt when L_{i+1} is being created



Lemma 1: $R^* \subseteq CC(s)$ ← Ex: Pf by induction.

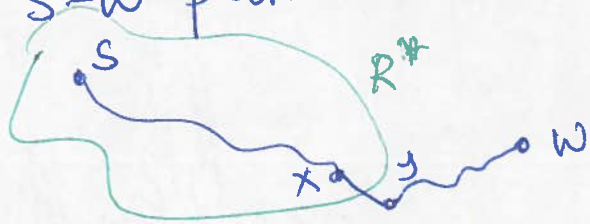
Lemma 2: $CC(s) \subseteq R^*$

Lemmas 1+2 ⟹ THEOREM

Pf idea for Lemma 2: By contradiction.

Assume $CC(s) \not\subseteq R^* \iff \exists\ w \in CC(s)$ BUT $w \notin R^*$

$\iff \exists$ a s-w path P but $w \notin R^*$



Since P starts off inside R^* & ends up outside R^*

$\exists\ (x, y) \in E$ s.t. $x \in R^*$, $y \notin R^*$ (Note $y = w$ is ok)

But then y is a candidate to be added to R

i.e. algo cannot terminate at that pt.

⟹ contradicts algo outputs R^* (& hence has terminated) ∎