Sep 29

BFS(s) // G is in adj. list format

$O(n)$
0. $CC[s] = T$, $CC[w] = F \quad \forall w \neq s$
1. $i = 0$
2. $L_0 = \{s\}$ // Linked list

3. While $L_i \neq \phi$ } T_1: # times this loop runs
   - 3.1 $L_{i+1} = \phi$ // Linked list (new) } $O(1)$
   - 3.2 For $u \in L_i$ $\leq n$
     - For $(u,w) \in E$ $= n_u \leq n$
       - If $CC[w] = F$
         - $CC[w] = T$
         - Add w to L_{i+1} } $O(1)$
   - 3.2 $i{+}{+}$ } $O(1)$

times while, 1st for loop runs

T_{12}: # times algo gets here

T_{123}: # times algo gets here

$T_1 \leq T_{123}$
$T_{12} \leq T_{123}$

Total runtime $\leq O(n) + T_1 \cdot O(1) + T_{123} \cdot O(1) + T_{12} \cdot O(1)$

$\leq O(n) + T_{123} \cdot O(1) + T_{123} \cdot O(1) + T_{123} \cdot O(1)$

$= O(n) + O(T_{123})$.

Analysis # 1: Obs: Each $u \in V$ is in atmost one L_i $\Rightarrow T_1 \leq n$

$\Rightarrow T_{123} \leq n^3$ (as each 3 loop runs $\leq n$ times)

$\Rightarrow$ overall: $O(n) + O(n^3) = O(n^3)$.

Analysis: $T_{123} \le O(n^2)$.

Claim: $T_{12} \le n$

Obs. $T_{123} \le T_{12} * n$
$\le n^2$

Obs: Each vertex is in at most one loop &

T_{12} = total number of vertices in all layer

$\Rightarrow$ Overall: $O(n) + O(n^2) = O(n^2)$

Analysis: $T_{123} \le O(m)$

$$T_{123} = \sum_{i=1}^{T_1} \sum_{u \in L_i} n_u$$

Each vertex in $\le$ one layer $\rightarrow$ $\le \sum_{u \in V} n_u = 2m \le O(m)$

Input size $= \Theta(m+n)$

BFS is a linear time algo.

$\Rightarrow$ Overall $= O(n) + O(m) = O(m+n)$ ■