Oct 6

Interval Scheduling Problem

Input: n intervals. i^{th} interval: ~~st~~ start time $s(i)$, finish time $f(i)$

Convention: i^{th} interval: $[s(i), f(i))$

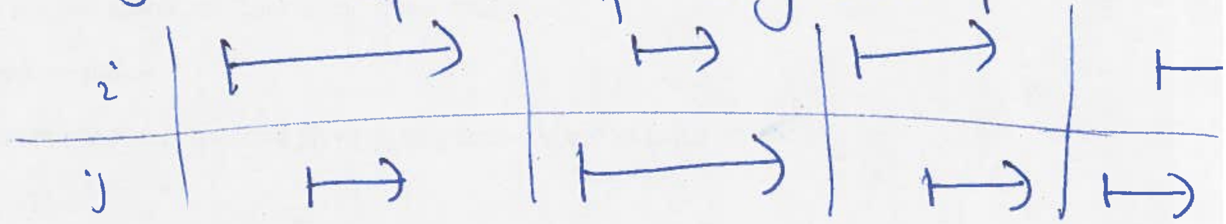
Ex: $[1,4) = \{1,2,3\}$ $\stackrel{def}{=} \{s(i), \dots, f(i)-1\}$

Output: A valid schedule with max # intervals.

Def: A schedule $S \subseteq [n]$

Def: A schedule S is valid if it has no conflicts

Def: i & j ~~ar~~ conflict if they overlap



Q: Given i & j, how quickly can you decide if i & j conflict?

A: $O(1)$. Check if one of the four cases above hold [each check: $O(1)$]

Obs: A valid schedule sorted by start time or finish time has the same order.

Assume: Input intervals are sorted by finish time

$f(1) \leq f(2) \leq f(3) \dots \leq f(n)$.

Greedy Algo

0. $R = [n]$ ($[n] \overset{def}{=} \{1, \dots, n\}$)
1. $S = \phi$
2. While ($R \neq \phi$)
   (2.1) Let i be the smallest index of any job in R
   (2.2) Add i to S
   (2.3) Delete i fr R
   (2.4) Delete all $j \in R$ that conflict wit i.
3. Return $S^* = S$.

THEOREM 1: S^* is an optimal valid schedule, ie. it has the max # of ~~possible~~ intervals in all possible valid schedules.

Ex 1: Algo terminates

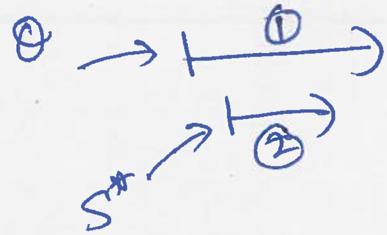
Ex 2: S^* is a valid schedule.

Pf. of correctness of greedy algo → "Greedy stays ahead
→ Exchange argumen (later)

Let Θ be an optimal solution (ie among all possible valid schedules, $|\Theta|$ is largest)

Ex 3: Convince yourself such on Θ always $\exists$.

Idea 1: $S^* = \mathcal{O}$ (Problem: Multiple optimal solution.



$n=2$

Idea: $|S^*| = |\mathcal{O}|$

THEOREM 2: $|S^*| = |\mathcal{O}|$

Crucial: Only things you can assume about $\mathcal{O}$:

(1) Valid schedule & (2) It is optimal.

NOTHING ELSE.