Oct 27

Minimum Spanning Tree (MST) problem

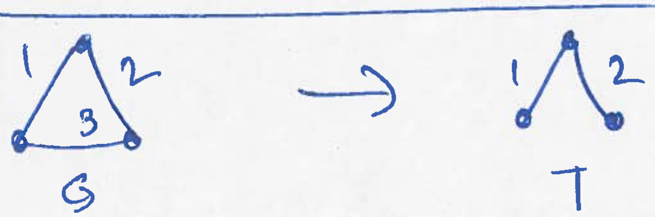
Input: $G = (V, E)$, $c_e \geq 0$ $\forall e \in E$ [this is for convenience]

$\uparrow$ G is connected.

Output: (i) $E' \subseteq E$ s.t. $T = (V, E')$ is connected

[Def: T is sub-graph]

(ii) minimize $c(T) = \sum_{e \in E'} c_e$.

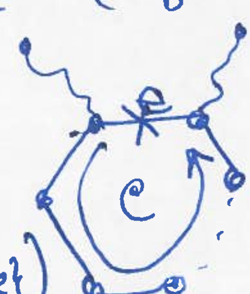


Prop: Let $c_e > 0$ $\forall e \in E$. Then the optimal solution T is a tree.

Pf (idea): By contradiction. Assume T is not a tree

$\Longrightarrow$ T has a cycle C

as T is connected | Let $e \in C$



$\rightarrow$ Delete e from T to get $T' = (V, E \setminus \{e\})$

Claim 1: T' is still connected

Claim 2: $c(T') < c(T)$

$\left.\right\}$ this contradicts the fact that T is an optimal solution.

$\hookrightarrow$ $c(T') = c(T) - c_e$

$< c(T)$ as $c_e > 0$ ■

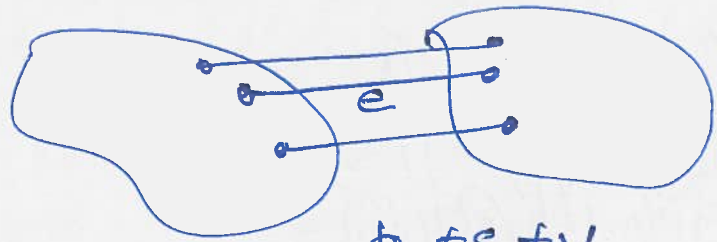
Brute Force algo: Go over all possibilities of $E' \subseteq E$ (2^m subsets) & check if (V, E') is connected. If so ~~keep~~ compute its cost & keep track of min cost.

CUT ~~RO~~ PROPERTY LEMMA

Assume: All c_e are distinct

$(S, V \setminus S)$ to be a cut.

$S \neq \phi$



$V \setminus S \neq \phi$

$\phi \neq S \neq V$

THEN: $\forall$ cuts $(S, V \setminus S)$, let e be the cheapest crossing edge $\Rightarrow$ e is in all MSTs of G.