$\boxed{\text{Sep 1}}$

Stable Matching/Marriage (non-feminist)

n women $W = \{w_1, \dots, w_n\}$
n men $M = \{m_1, \dots, m_n\}$

$\boxed{n=2}$ $M = \{BP, BBT\}$
$W = \{JA, AJ\}$

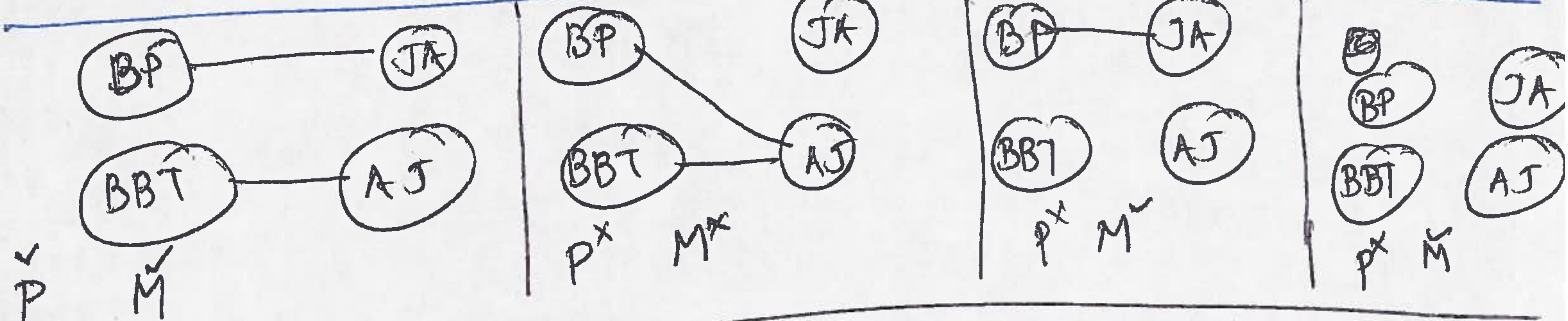
Def (Matching) A subset $S \subseteq M \times W \overset{def}{=} \{(m,w) \mid m \in M, w \in W\}$ is a matching if

(i) $\forall w \in W$, $\exists$ at most one man $m \in M$ s.t $(m,w) \in S$

$\boxed{\text{AND}}$

(ii) $\forall m \in M$, $\exists$ ______________ woman $w \in W$ s.t $(m,w) \in S$

Def (Perfect Matching) at most gets replaced by exactly



Aside! (Q2, HW0) $n!$ perfect matchings on n men & n women

Def (Preference List) $\forall w \in W$, L_w: total ranking of all $m \in M$
$\forall m \in M$, L_m: ______________ $w \in W$

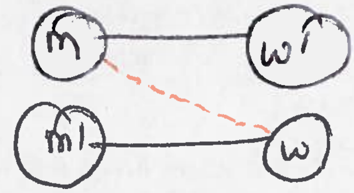
Ex: L_{BP}: AJ > JA | L_{JA}: BP > BBT
L_{BBT}: AJ > JA | L_{AJ}: BP > BBT

→ $2n$ total preference lists
→ $2n^2$ elements in all preference lists

Def (Stable matching) A stable S is (1) a perfect matching and (2) has NO instability

Def (Instability) Given a perfect matching S, $2n$ preference list

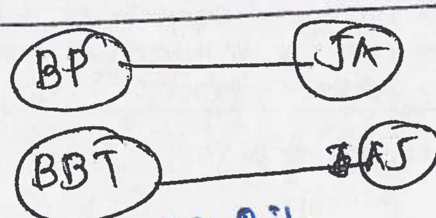
a pair $(m, w) \notin S$ is an instability if



(1) $w > w'$ in L_m

(2) $m > m'$ in L_w

($n=2$ pref list)



(BBT, JA) instability

AND

(BP, AS) is an instability

SMP: Input: M, W & $2n$ preference lists

$|M| = |W| = n$

Output: A stable matching (if $\exists$ one)

Q: Design algo to solve problem above in <u>finite</u> time.