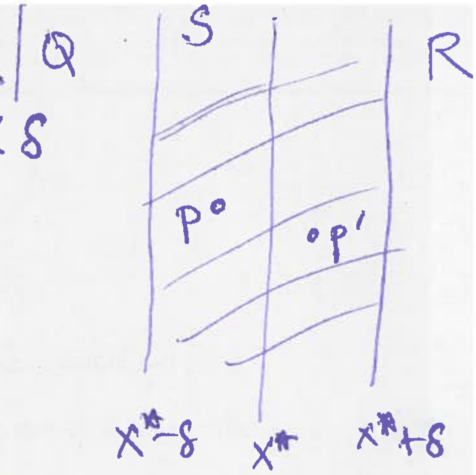
Nov 13

KICKASS PROPERTY LEMMA

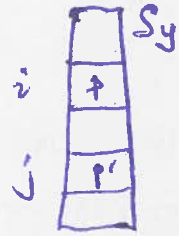


For every $p \neq p' \in S$ s.t. $d(p, p') < \delta$

s.t. $S_y[i] = p$

$S_y[j] = p'$ $\{ j > i \}$

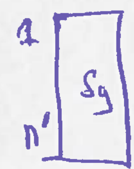
$\Rightarrow |i - j| \leq 15$



NOTE: (i) You can make 15 to be 9 (Ex.) Can be as small as 7.

(ii) Compute S_y from P_y in $O(n)$ time

Q: How does the Kickass Property Lemma imply the $O(n)$ implementation of Closest-in-Box?



A: Closest-in-Box (S_y, δ)

for $i = 1 .. n'$

Let (p_i, p_i') be the closest pair of points among $(S_y[i], S_y[i+1]), (S_y[i], S_y[i+2]), \ldots$

$\ldots (S_y[i], S_y[\min(i+15, n')])$

Let ~~Let~~ (p, p') be the closest pair of points among (p_i, p_i') for $i = 1 .., n'-1$

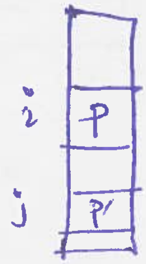
If $d(p, p') < \delta$

return (p, p')

else return null.

Pf (idea) of Kickass Property Lemma:

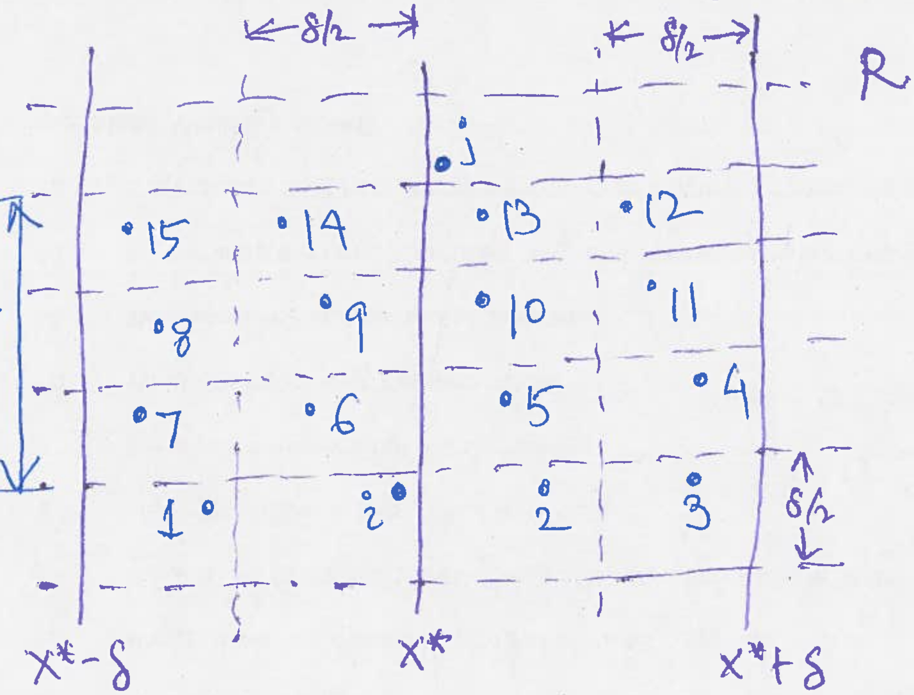
For contradiction, assume $|i - j| \geq 16$



S_y

$d(p,p') < \delta$ (1)

Q



$\frac{3\delta}{2}$

$\Downarrow$

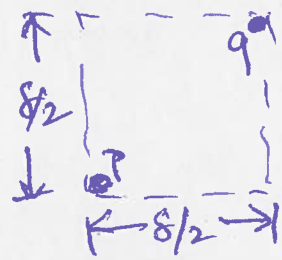
$d(p,p') > \frac{3\delta}{2} > \delta$

$\Downarrow$

contradicts (1) ■

Claim: Every $\frac{\delta}{2} \times \frac{\delta}{2}$ square has at most one point in it.

Pf (idea): Assume p & q are in the same square



Ex: The ~~furthest~~ pairs are at the diagonal corners

$\Rightarrow d(p,q) \leq \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}} = \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta.$

BUT square completely inside Q or R $\Rightarrow$ contradicts definition of δ ■