Sep 11

LEMMA 2: The output of GS algo (S) is a perfect matching

OBS 0 S is a matching

OBS 1 Once a man gets engaged, he keeps on getting engaged to better women.

OBS 2 If w proposes to m after m' $\Rightarrow$ m' > m in L_w

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LEMMA 4: If at the end of an iteration, w is free $\Rightarrow$ w has not proposed to all men

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Pf of LEMMA 2: **Pf. idea:** Pf by contradiction (use Obs 0, lemma 4, algo definition)

Pf. details: Assume for sake of contradiction that S is NOT a perfect matching.

$\Rightarrow$ $\exists$ a free woman w ——— (1)

(By Obs 0 + algo def^n)

$\Rightarrow$ w has not proposed to all men. ——— (2)

(by LEMMA 4)

$\Rightarrow$ $\exists$ man m that w has not proposed to yet ——— (*)

Since algo has terminated

$\Rightarrow$ there is no free woman who has NOT proposed to all men $\Rightarrow$ contradicts (*). ∎

(cond^n of loop)

Pigeon-hole principle: If $\leq n-1$ pigeons are put into n holes $\Rightarrow$ $\exists$ atleast one empty hole.

Pf of LEMMA 4: Pf: Pf by contradiction (Pigeon-hole principle + OBS 1)

Pf idea

Pf details: For sake of contradiction, assume ∃ a free woman who has proposed to all men.

⇒ all n men are engaged. — (#)

(OBS 1 + Algo statement)

Since w is free ⇒ ≤ n-1 women are engaged.

⇒ ≤ n-1 men are engaged

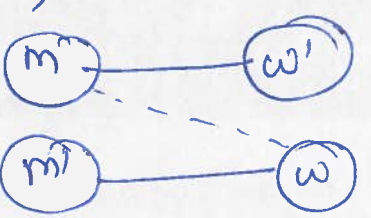
by the pigeonhole principle ⇒ contradicts (#) ■

(hole :: men
pigeon :: women
assignment :: engaged)

LEMMA 3: S has no instability

Pf idea: Pf by contradiction with a case analysis (Obs 1, Obs 2, LEMMA 2)

Pf-details: Assume S has an instability. Since by Lemma 2, S is a perfect matching, we have a pair (m, w) ∉ S



⇒ (i) w > w' in Lm AND

(ii) m > m' in Lw.

Case 1: w never proposed to m but (m', w) are engaged ⇒ w proposed to m'

⇒ by (Obs 2) m' > m in Lw ⇒ contradicts (ii)

Case 2: w proposed to m

Case 2.1: (m, w) got engaged then. But (m, w') are engaged. ⇒ w' > w in Lm ⇒ contradicts (i)

Obs 1

Case 2.2: (m, w'') were engaged & w'' > w in Lm

⇒ (by Obs 1) w' > w'' in Lm. — ⊕ ⇒ w' > w in Lm ⇒ contradicts (i) ■