Sep 15

Aside: While USUALLY we will use asymptotic analysis to talk about runtime of an algo, defns are INDEPENDENT of semantics of $g(N)$.

Ex: $g(N)$ = denote # times Akash says no to me in his N^{th} month of existence.

$T_A(N)$ = max # steps A takes on ANY input of size N.

A ← algo, N ← input size

↳ Talk about $T_A(N)$ is $O(g(N))$ or it is $\Omega(f(N))$

Example: Search problem: $\boxed{N = n+1}$

i/p: $a_0, \ldots, a_{n-1}; v$

o/p: i s.t. $a_i = v$ (if one such i $\exists$)

-1 o/w.

SEARCH($a_0, \ldots, a_{n-1}; v$)

$T_0 \cdot T_1$

```
for (i=0..n-1)          <- T0: # time this loop runs <= n is O(
    if a_i == v
        return i;       } T1: runtime of body of loop  O(1)
return -1;              <- T2: time taken for this step  O(1)
```

$T_{SEARCH}(N) \leq T_0 \cdot T_1 + T_2 \leq \underbrace{n \cdot O(1)}_{\text{mult}} + O(1)$ $\leq O(n)$

$\leq \underbrace{O(n) + O(n)}_{\text{additive}} \leq O(n)$ ■

CLAIM 1: $T_{SEARCH}(N)$ is $O(N)$.

CLAIM 2: $T_{SEARCH}(N)$ is $\Omega(N)$. ($\Rightarrow$ it is $\Theta(N)$)

Recall: $T(N)$ is max # steps taken by any i/p of size N

$$= \max \{ \text{\_\_\_\_ on } 1^{st} \text{\_\_\_\_}, \text{\_\_\_\_ on } 2^{nd} \text{\_\_\_\_}, \text{\_\_\_\_ on } 3^{rd} \text{\_\_\_\_}, \ldots \}$$

more generally max of a set of #s.

Pf idea/strategy: Exhibit one i/p of size N ~~tod~~ ~~too~~ on which algo takes $\geq L$ steps (SEARCH: $L = \Omega(N)$)
$\Rightarrow T(N) \geq L$.

Pf details: ~~Con~~ Fix $n \geq 1$.
Consider $\underset{0 \leq i < N}{a_i = 2^i}$, $v = n$ (many other possible e.g. $a_{n-1} = v$ & $a_i \neq v$ $\forall i \leq n-$

Runtime or $T(N) \geq T_0 \cdot T_1 \geq n \cdot 1 = n \geq \Omega(n)$
$= \Omega(N)$

"Best case analysis" $a_0 = v \Rightarrow T(N) \geq \Omega(1)$.

Implement GS algo:

$\boxed{\text{Initialization}} \leftarrow T_0$

while $(\ldots)$ $\leftarrow$ T_1 = # iterations $\leq n^2$

$\boxed{\text{Body of loop}} \leftarrow T_2$

$\boxed{\text{Output } S} \leftarrow T_3$

If we argue
$T_0, T_3 \leq O(n)$
$\boxed{T_2 \leq O(1)}$
non-obvious

$$T(N) \leq T_0 + T_1 \cdot T_2 + T_3$$
$$\leq O(n) + n^2 \cdot O(1) + O(n)$$
$$\leq O(n) + O(n^2) \leq O(n^2) + O(n^2) \leq O(n^2) \; \blacksquare$$