

Step 15 Aside: While USUALLY we will use asymptotic analysis to talk about runtime of an algo, defs are INDEPENDENT of semantics of  $g(N)$ .

Ex:  $g(N)$  = denote # times Atash says no to me in his  $N$ th month of existence.

$T_A(N)$  = max # steps A takes on ANY input of size  $N$ .

algo  $\rightarrow$   $T_A(N)$   $\uparrow$  input size

$\hookrightarrow$  Talk about  $T_A(N)$  is  $O(g(N))$  or it is  $\Omega(f(N))$

Example: Search problem:  $N = n+1$

i/p:  $a_0, \dots, a_{n-1}; v$   
 o/p:  $i$  s.t.  $a_i = v$  (if one such  $i \exists$ )

SEARCH ( $a_0, \dots, a_{n-1}; v$ )

$T_0 \cdot T_1$  { for ( $i=0 \dots n-1$ )  $\leftarrow T_0$ : # time this loop runs  $\leq n$  is  $O(1)$   
 if  $a_i == v$  }  $T_1$ : runtime of body of loop  $\approx O(1)$   
 return  $i$ ; }  
 return -1;  $\leftarrow T_2$ : time taken for this step  $\approx O(1)$

$$T_{SEARCH}(N) \leq T_0 \cdot T_1 + T_2 \leq n \cdot O(1) + O(1) \leq O(n)$$

$$\leq \underbrace{O(n)}_{\text{mult}} + O(n) \leq O(n) \quad \text{additive}$$

CLAIM 1:  $T_{SEARCH}(N)$  is  $O(N)$ .

CLAIM 2:  $T_{SEARCH}(N)$  is  $\Omega(N)$ . ( $\Rightarrow$  it is  $\Theta(N)$ )

Recall:  $T(N)$  is max # steps taken by any i/p of size  $N$   
 $= \max \left\{ \begin{array}{l} \text{_____ on 1st} \\ \text{_____ on 2nd} \\ \text{_____ on 3rd} \end{array} \right\}$

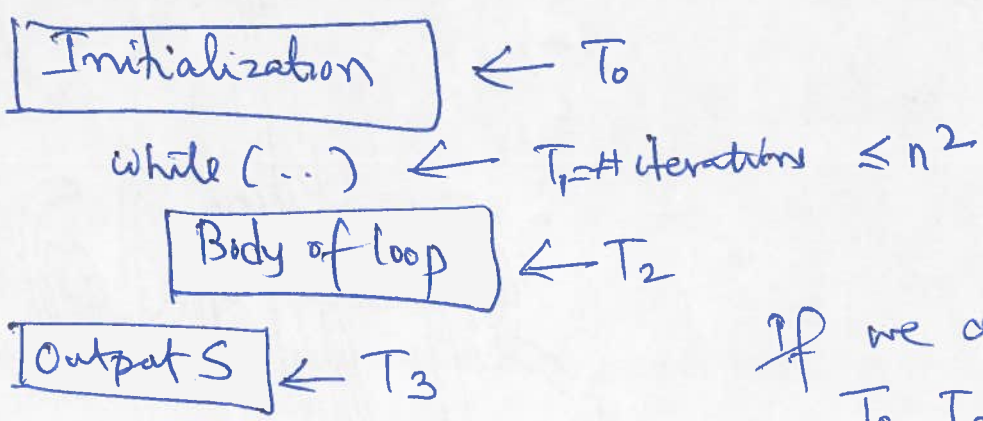
more generally max of a set of #s.  $\left. \begin{array}{l} \vdots \\ \vdots \end{array} \right\}$

Pf idea/strategy: Exhibit one i/p of size  $N$  ~~that~~ on which algo takes  $\geq L$  steps (SEARCH:  $L = \Omega(N)$ )  
 $\Rightarrow T(N) \geq L$ .

Pf details: ~~Fix~~ fix  $n \geq 1$ .  
 Consider  $0 \leq i < n$   $a_i = 2$ ,  $v = n$  (many other possible e.g.  $a_{n-1} = v$  &  $a_i \neq v \forall i < n$ )  
 Runtime or  $T(N) \geq T_0 \cdot T_1 \geq n \cdot 1 = n \geq \Omega(n) = \Omega(N)$

"Best case analysis"  $a_0 = v \Rightarrow T(N) \geq \Omega(1)$ .

Implement GS algo:



If we argue  $T_0, T_3 \leq O(n)$   
 $T_2 \leq O(1)$   
*non-obvious*

$$\begin{aligned}
 T(N) &\leq T_0 + T_1 \cdot T_2 + T_3 \\
 &\leq O(n) + n^2 \cdot O(1) + O(n) \\
 &\leq O(n) + O(n^2) \leq O(n^2) + O(n^2) \leq O(n^2)
 \end{aligned}$$