

Lecture 17

CSE 331

Oct 5, 2018

Homework 5

Homework 5

Due by **11:59pm, Thursday, October 11, 2018**.

Make sure you follow all the [homework policies](#).

All submissions should be done via [Autolab](#).

Sample Problem

The Problem

Extend the topological ordering algorithm we saw in class so that, given an input directed graph G , it outputs one of two things: (a) a topological ordering, thus establishing that G is a DAG, or (b) a cycle in G , thus establishing that G is not a DAG.

The running time of your algorithm should be $O(m + n)$ for a directed graph with n nodes and m edges.

[Click here for the Solution](#)

Solutions to HW 4

End of the lecture

Quiz 1 on Monday

note ☆ stop following 125 views

Quiz 1 on Monday, Oct 8

The first quiz will be from 8-8:10am in class on **Monday, October 8**. We will have a 5 mins break after the quiz and the lecture will start at 8:15am.

We will hand out the quiz paper at 7:55am but you will **NOT** be allowed to open the quiz to see the actual questions till 8am. However, you can use those 5 minutes to go over the instructions and get yourself in the zone.

There will be two T/F with justification questions (like those in the sample mid term 1: @458.)
#pin

quiz1

edit good note 0 Updated 3 days ago by Mark Armstrong and Abri Rudra

Done with mid-term material

note stop following 83 views

Linear time topological ordering

As I mentioned in the lecture today, one can actually implement TopOrd in $O(m + n)$ time instead of the $O(n^2)$ analysis that we did in the lecture today. The details are in the book or you can watch [this video from last year](#).

BTW this is where the mid-term material ends: so all of Chapters 1, 2 and 3 in the textbook (except Chapter 1.2).

#pin

lectures

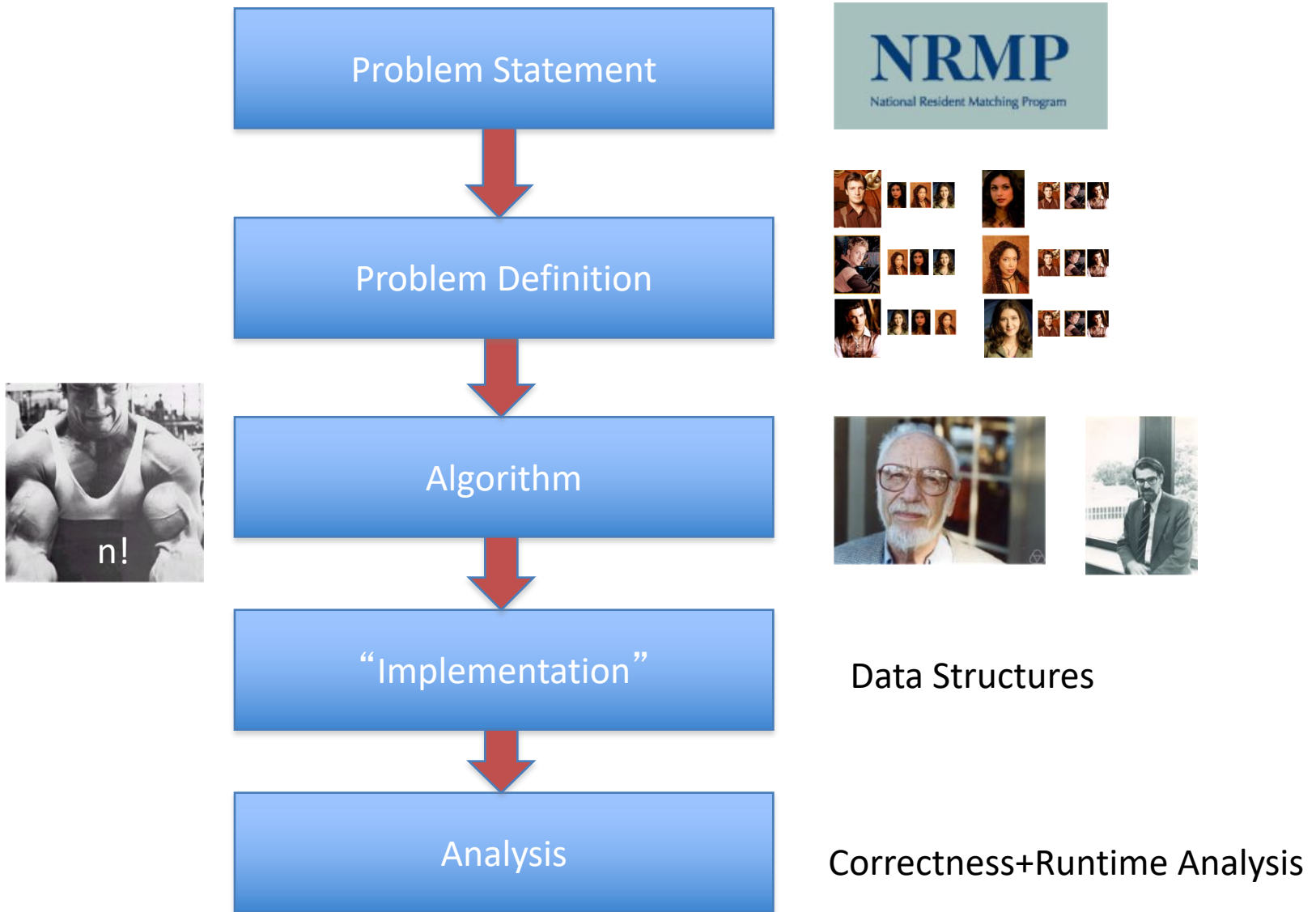
edit good note 0 Updated 1 day ago by Atri Rudra

Mid-term material

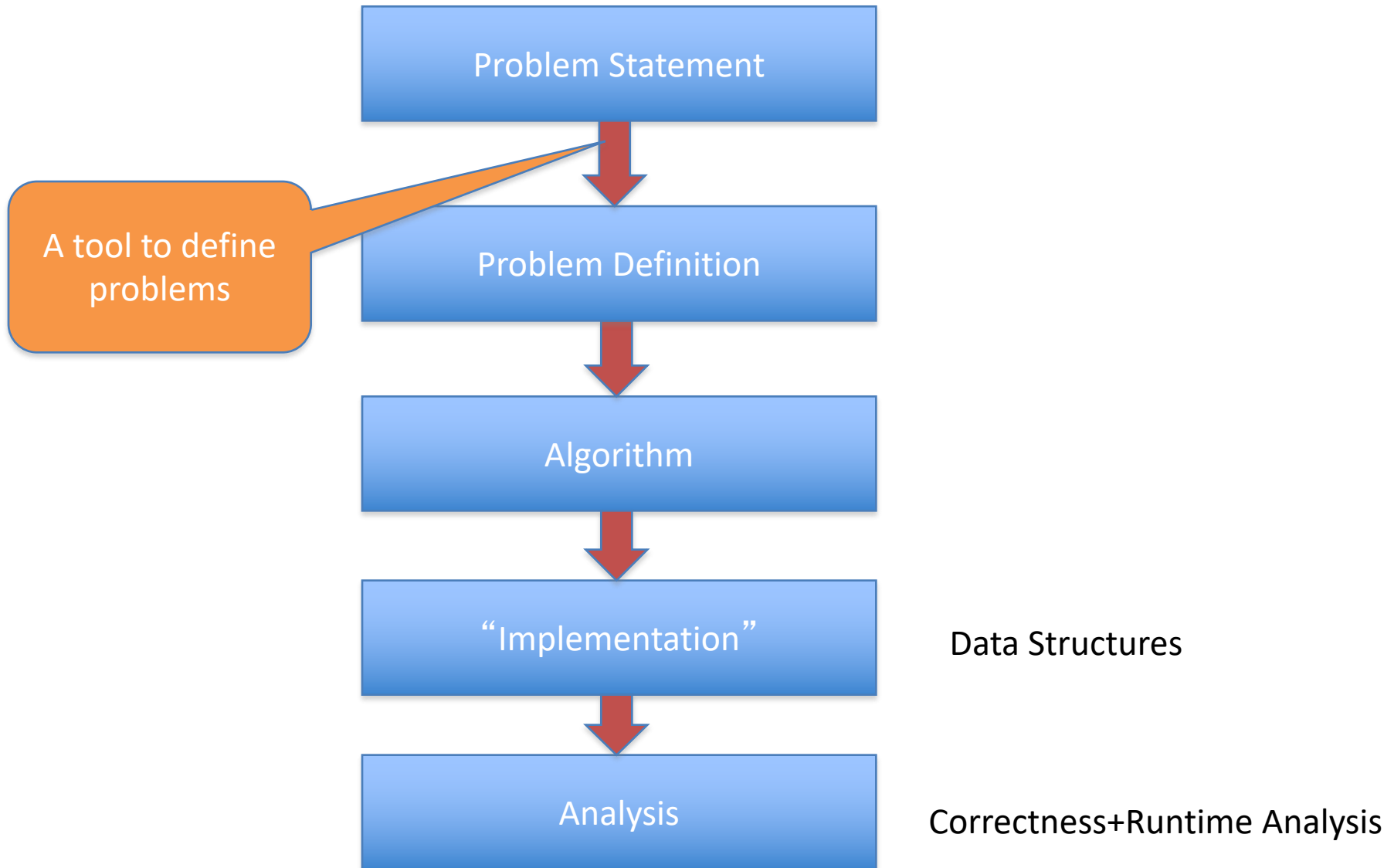
Everything we have covered so far (essentially Chaps 1-3 except Sec 1.2)

See piazza post on how to prepare for the mid-terms

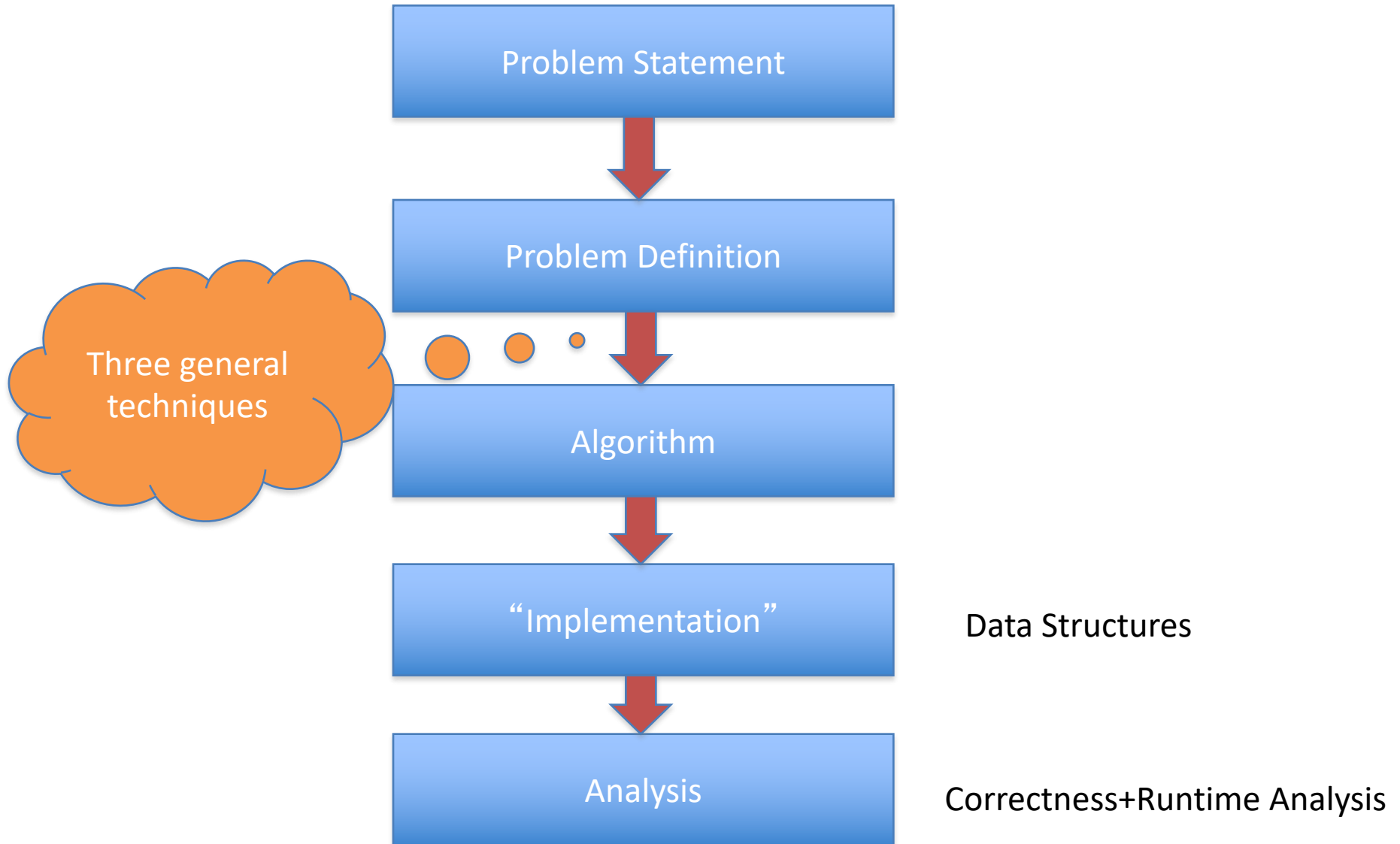
Main Steps in Algorithm Design



Where do graphs fit in?



Rest of the course



Greedy algorithms

Build the final solution piece by piece

Being short sighted on each piece

Never undo a decision

Know when you see it



End of Semester blues

Can only do one thing at any day: what is the maximum number of tasks that you can do?



Write up a term paper

Party!

Exam study

Homework

331 HW

Project

Sunday

Monday

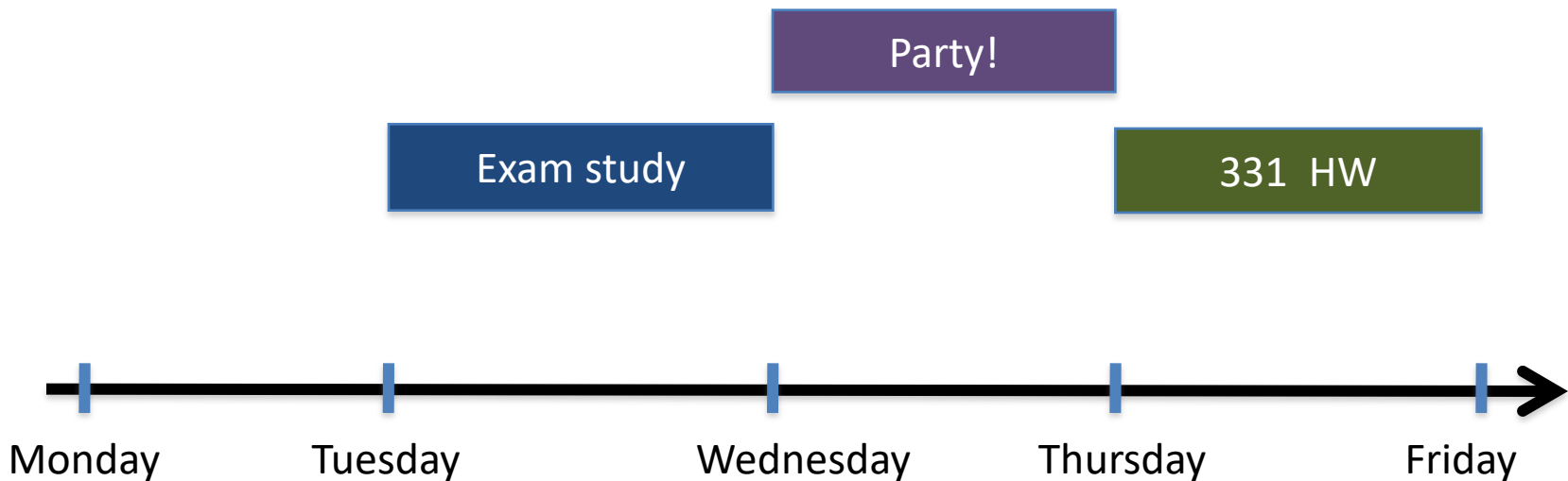
Tuesday

Wednesday

Thursday

The optimal solution

Arrange tasks in some order and iteratively pick non-overlapping tasks



Interval Scheduling Problem

Input: n intervals $[s(i), f(i))$ for $1 \leq i \leq n$



$\{s(i), \dots, f(i)-1\}$

Output: A schedule S of the n intervals

No two intervals in S conflict

$|S|$ is maximized

Algorithm with examples

Interval Scheduling via examples

In which we derive an algorithm that solves the Interval Scheduling problem via a sequence of examples.

The problem

In these notes we will solve the following problem:

Interval Scheduling Problem

Input: An input of n intervals $[s(i), f(i))$, or in other words, $\{s(i), \dots, f(i) - 1\}$ for $1 \leq i \leq n$ where i represents the intervals, $s(i)$ represents the start time, and $f(i)$ represents the finish time.

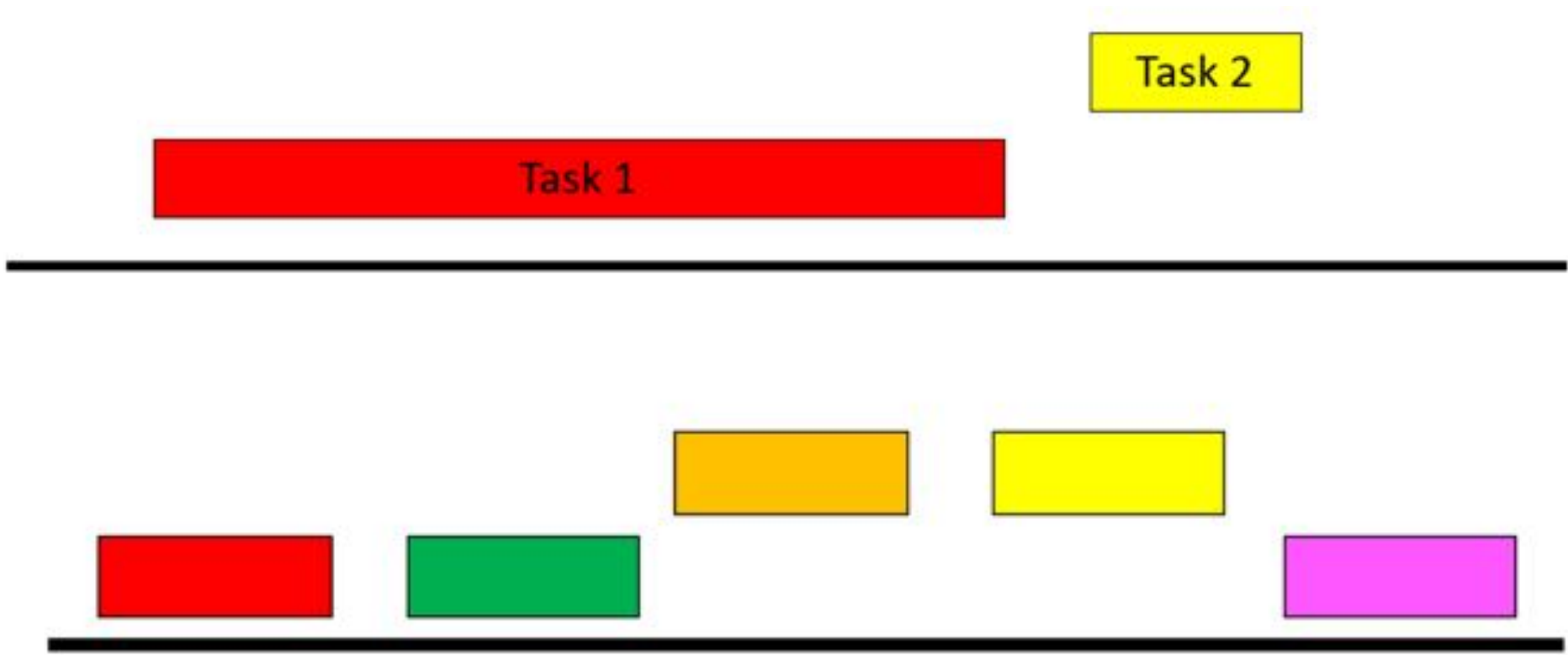
Output: A schedule S of n intervals where no two intervals in S conflict, and the total number of intervals in S is maximized.

Sample Input and Output

Input:

Example 1

No intervals overlap



Algorithm?



No intervals overlap

R : set of requests

Set S to be the empty set

While R is not empty

 Choose i in R

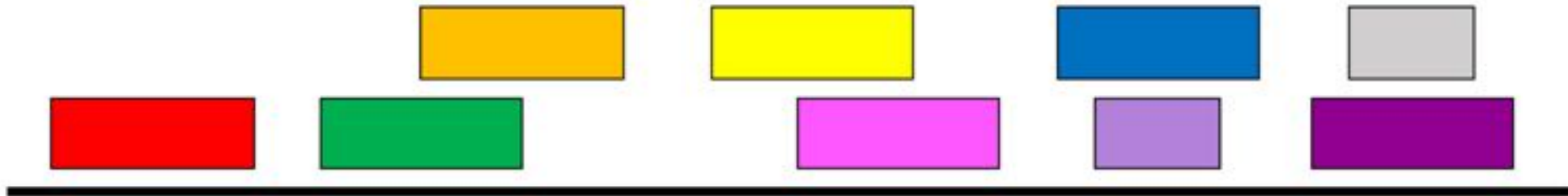
 Add i to S

 Remove i from R

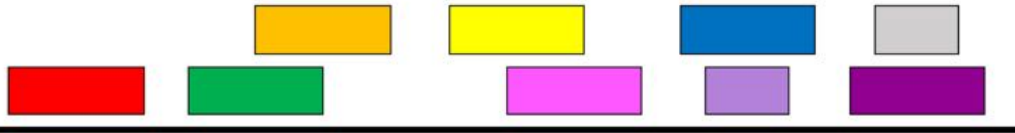
Return $S^* = S$

Example 2

At most one overlap



Algorithm?



At most one overlap

R : set of requests

Set S to be the empty set

While R is not empty

 Choose i in R

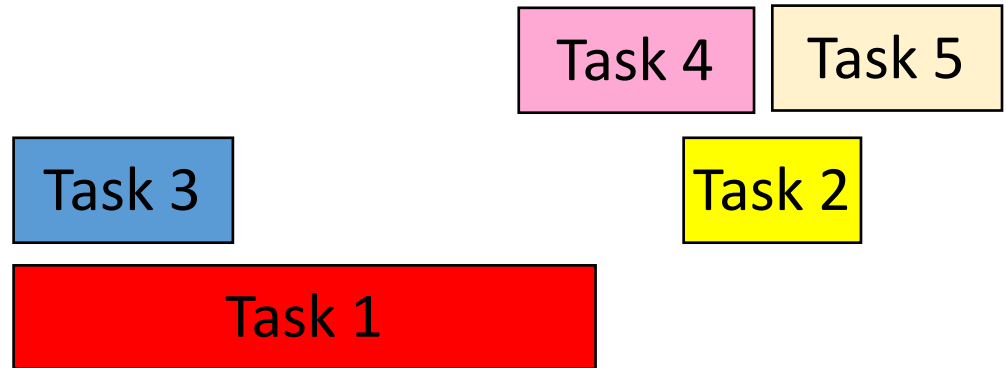
 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

Example 3

More than one conflict



Set S to be the empty set

While R is not empty

 Choose i in R

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

Greedily solve your blues!

Arrange tasks in some order and iteratively pick non-overlapping tasks



Write up a term paper

Party!

Exam study

331 HW

Project

Monday

Tuesday

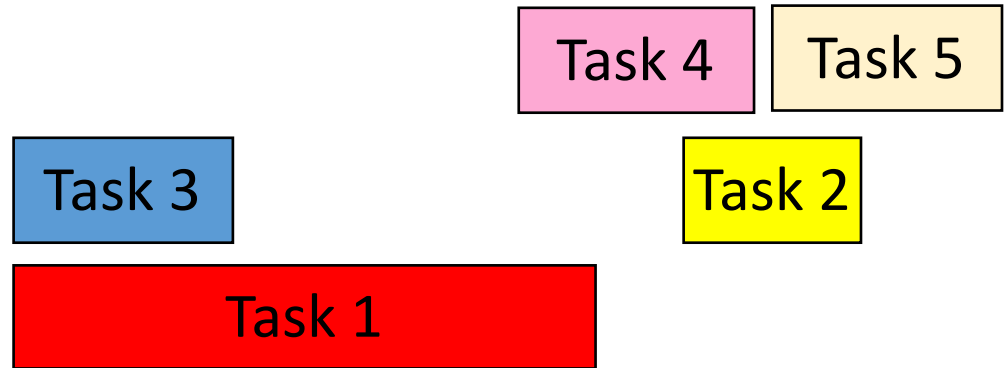
Wednesday

Thursday

Friday

Making it more formal

More than one conflict



Set S to be the empty set

While R is not empty

Choose i **in** R that minimizes $v(i)$

 Add i to S

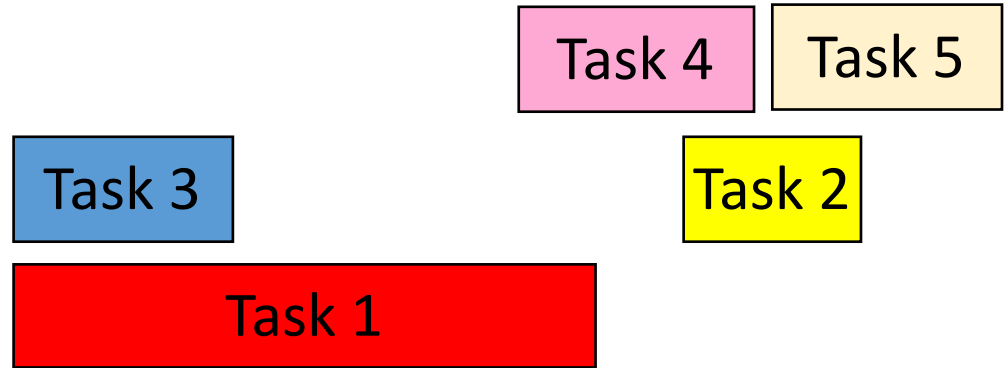
 Remove all tasks that conflict with i from R

Return $S^* = S$

Associate a
value $v(i)$
with task i

What is a good choice for $v(i)$?

More than one conflict



Set S to be the empty set

While R is not empty

 Choose i in R that minimizes $v(i)$

 Add i to S

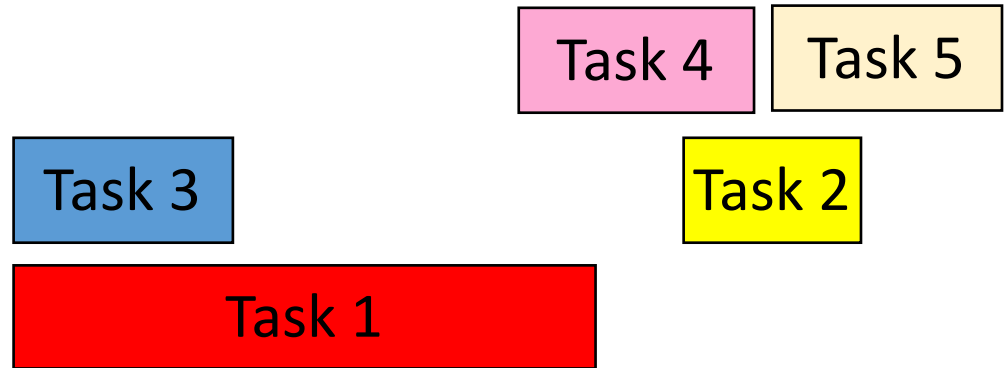
 Remove all tasks that conflict with i from R

Return $S^* = S$

Associate a
value $v(i)$
with task i

$$v(i) = f(i) - s(i)$$

Smallest duration first



Set S to be the empty set

While R is not empty

 Choose i in R that minimizes $f(i) - s(i)$

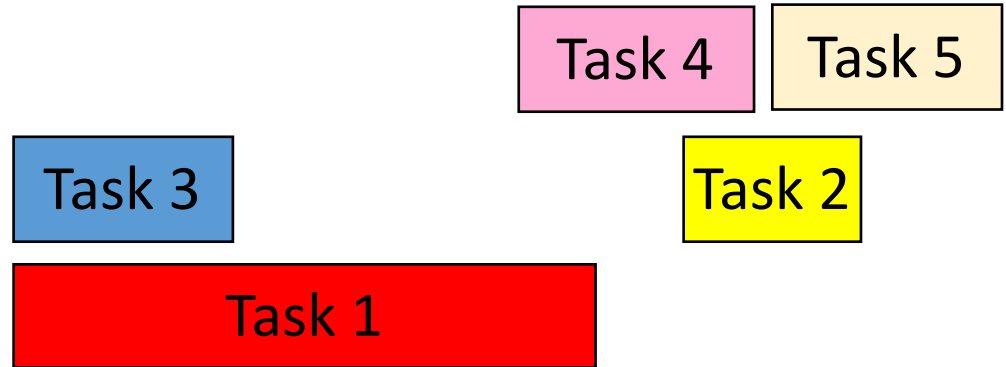
 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

$$v(i) = s(i)$$

Earliest time first?



Set S to be the empty set

While R is not empty

 Choose i in R that minimizes $s(i)$

 Add i to S

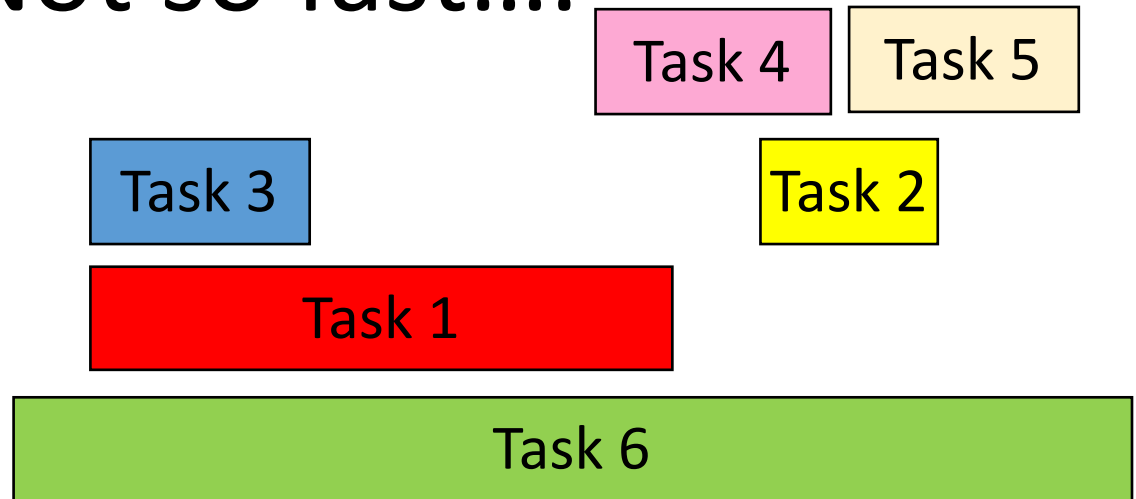
 Remove all tasks that conflict with i from R

Return $S^* = S$

So are we
done?

Not so fast....

Earliest time first?



Set S to be the empty set

While R is not empty

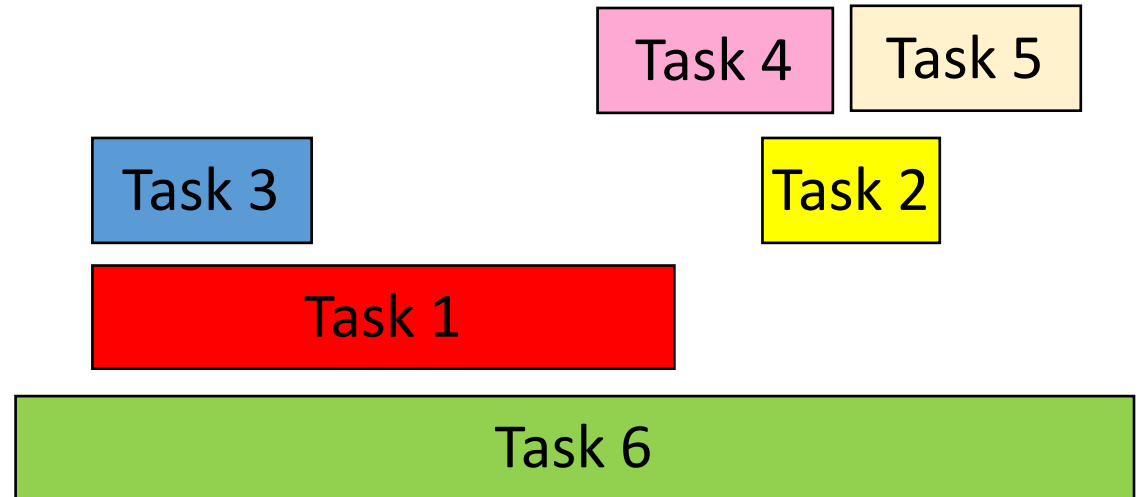
 Choose i in R that minimizes $s(i)$

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

Pick job with minimum conflicts



Set S to be the empty set

While R is not empty

 Choose i in R that has smallest number of conflicts

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

So are we
done?

Nope (but harder to show)

Set S to be the empty set

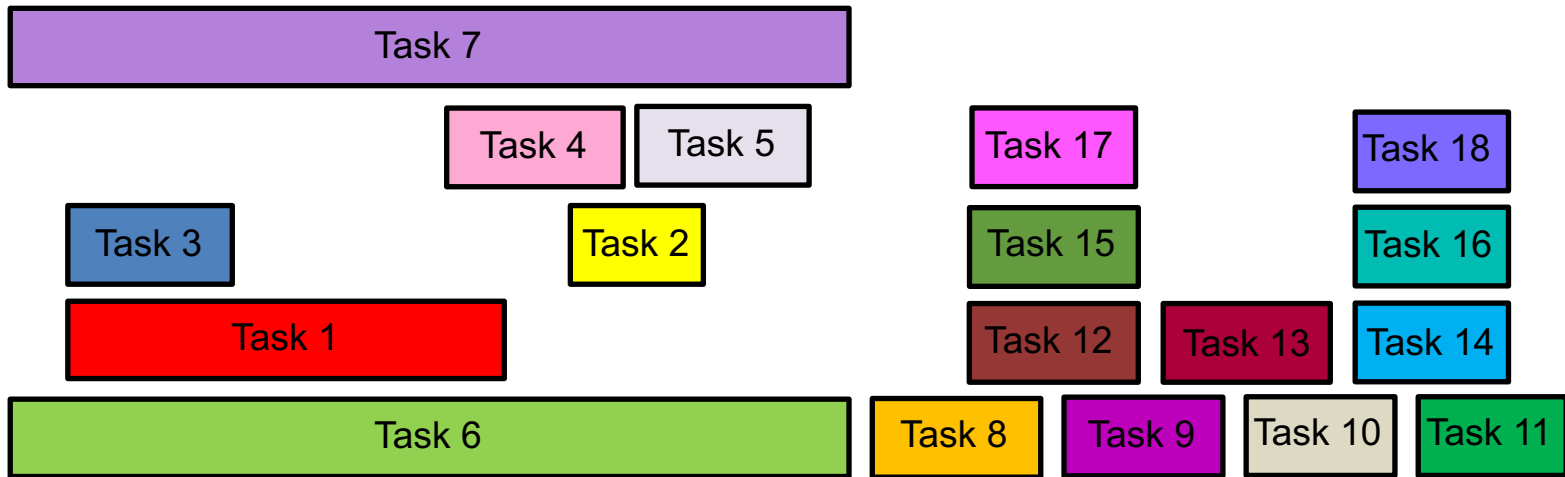
While R is not empty

 Choose i in R that has smallest number of conflicts

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$



Set S to be the empty set

While R is not empty

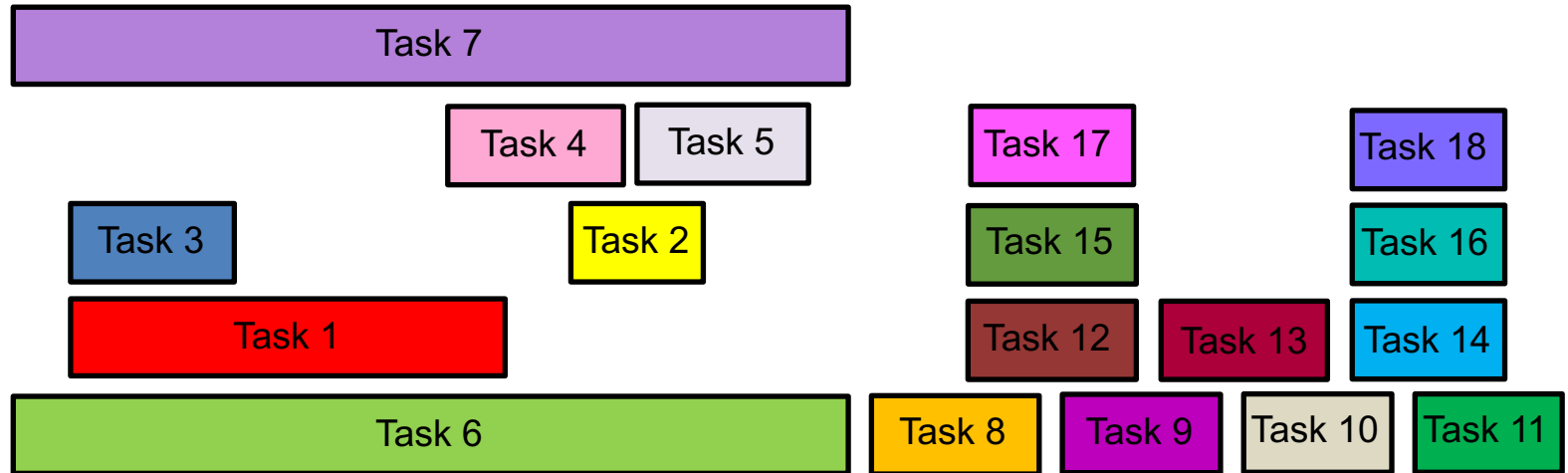
 Choose i in R that has smallest number of conflicts

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

Algorithm?



Set S to be the empty set

While R is not empty

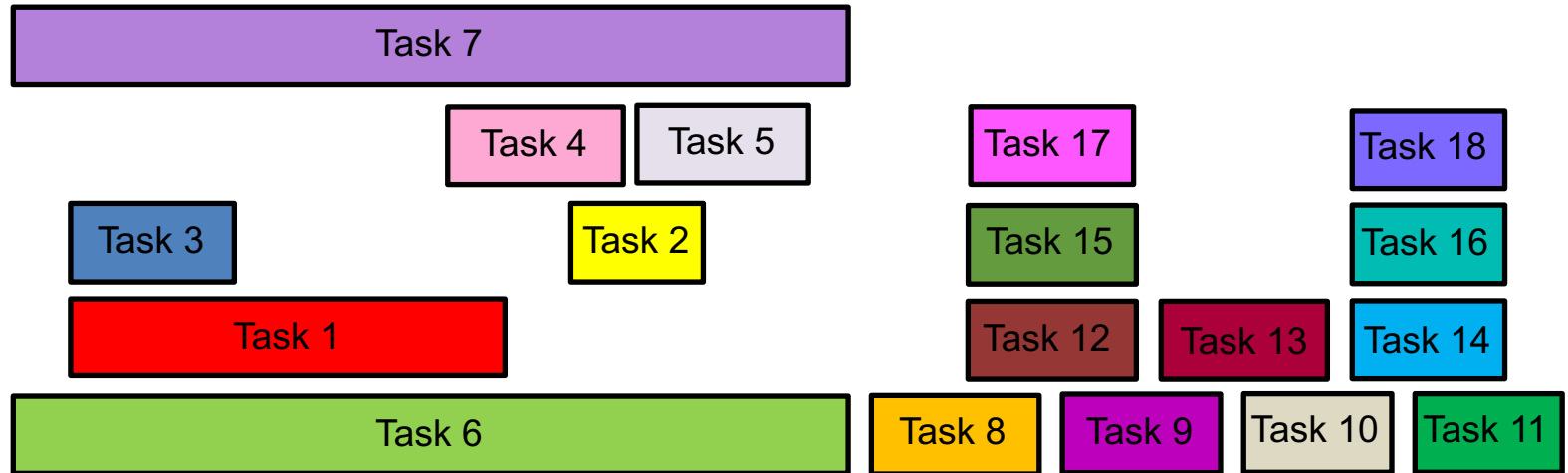
 Choose i in R that minimizes $v(i)$

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

Earliest finish time first



Set S to be the empty set

While R is not empty

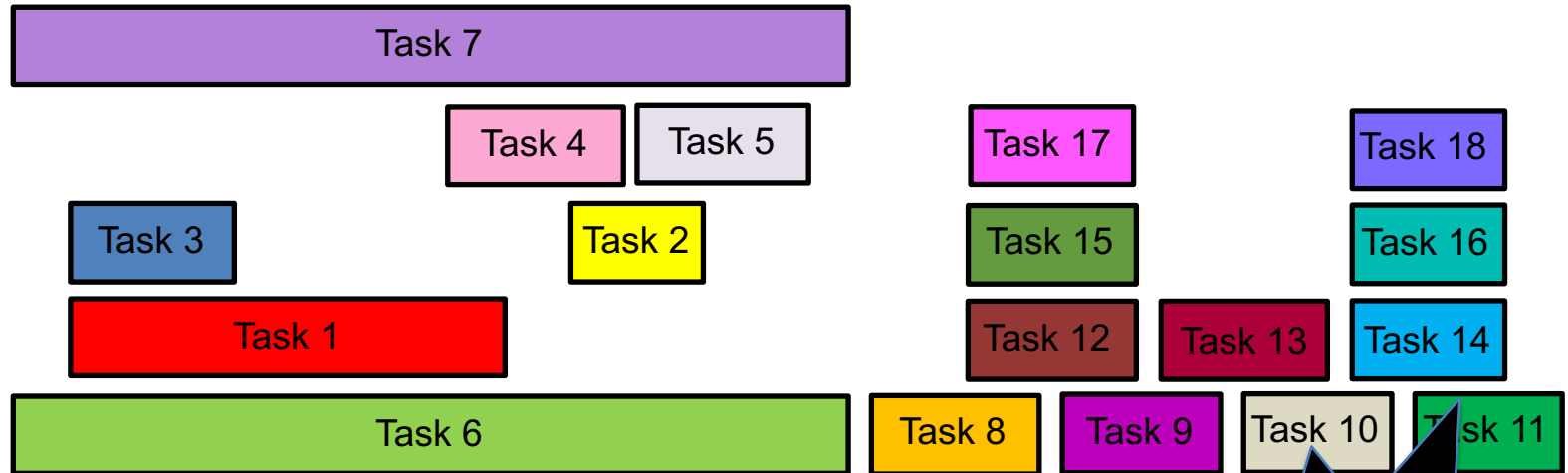
 Choose i in R that minimizes $f(i)$

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

Find a counter-example?



Set S to be the empty set

While R is not empty

 Choose i in R that minimizes $f(i)$

 Add i to S

 Remove all tasks that conflict with i from R

Return $S^* = S$

It
works!

Questions?



Today's agenda

Prove the correctness of the algorithm

Final Algorithm

R : set of requests

Set S to be the empty set

While R is not empty

 Choose i in R with the earliest finish time

 Add i to S

 Remove all requests that conflict with i from R

Return $S^* = S$