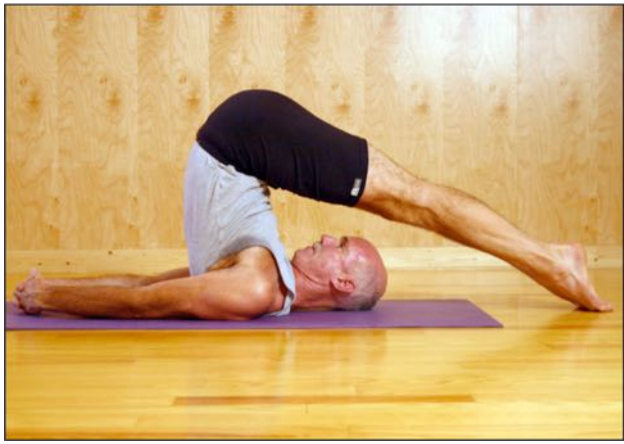
Lecture 31

CSE 331

Nov 13, 2018

Counting Inversions



Input: n distinct numbers $a_1,a_2,\ldots,a_n$

Inversion: (i,j) with $i < j$ s.t. $a_i > a_j$

Output: Number of inversions

Divide and Conquer

Divide up the problem into at least two sub-problems

Recursively solve the sub-problems

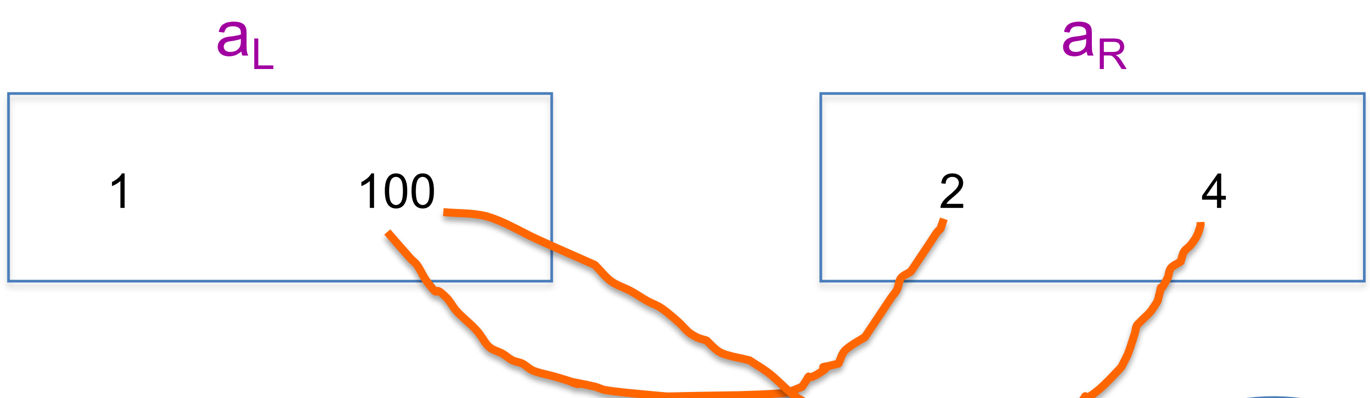
Solve all sub-problems: Mergesort

Solve some sub-problems: Multiplication

Solve stronger sub-problems: Inversions

"Patch up" the solutions to the sub-problems for the final solution

Handling crossing inversions



Why should a_L and a_R be sorted?

http://www.dovecoteidea.com/

Sort a_L and a_R recursively!

Mergesort-Count algorithm

Input: $a_1, a_2, \ldots, a_n$

Output: Numbers in sorted order+ #inversion

```
MergeSortCount( a, n )
    If n = 1 return ( 0 , a_1)
    If n = 2 return ( a1 > a2, min(a_1,a_2); max(a_1,a_2))
    a_L = a_1,..., a_{n/2}        a_R = a_{n/2+1},..., a_n
    (c_L, a_L) = MergeSortCount(a_L, n/2)
    (c_R, a_R) = MergeSortCount(a_R, n/2)
    (c, a) = MERGE-COUNT(a_L,a_R)
    return (c+c_L+c_R,a)
```

Counts #crossing-inversions+ MERGE

$T(2) = c$

$T(n) = 2T(n/2) + cn$

O(n log n) time

O(n)

MERGE-COUNT(a_L,a_R)

$a_L = l_1,\ldots, l_{n'}$ $\qquad$ $a_R = r_1,\ldots, r_m$

$c = 0$

$i,j = 1$

while $i \leq n'$ and $j \leq m$

    if $l_i < r_j$

        i ++

        add l_i to output

    else

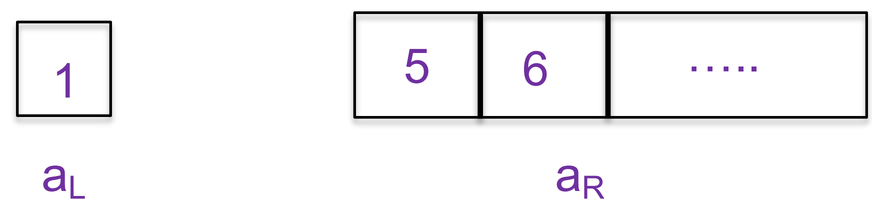
        add r_j to output

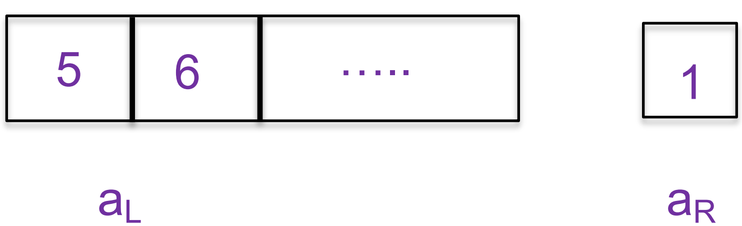
        j ++

        c += $n' - i + 1$

Output any remaining items

return c



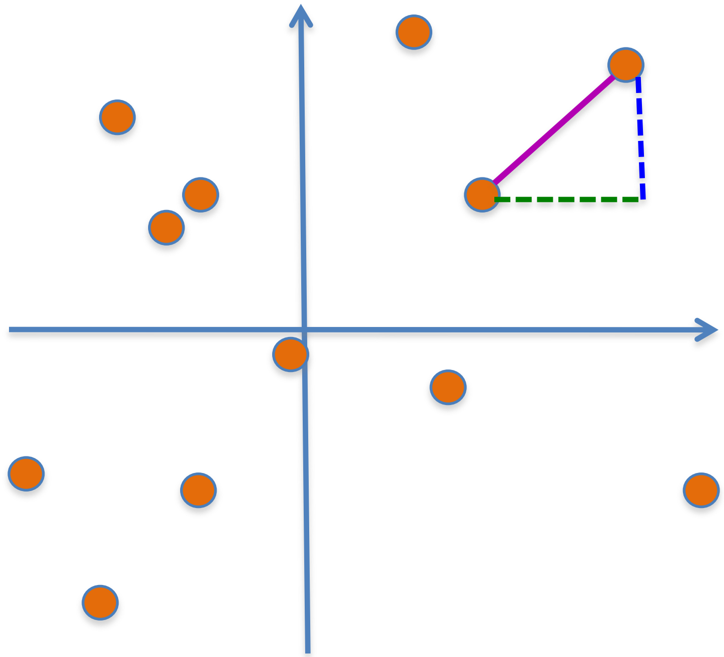


Closest pairs of points

Input: n 2-D points $P = \{p_1,\ldots,p_n\}$; $p_i=(x_i,y_i)$

$$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$$

Output: Points p and q that are closest



Group Talk time

$O(n^2)$ time algorithm?

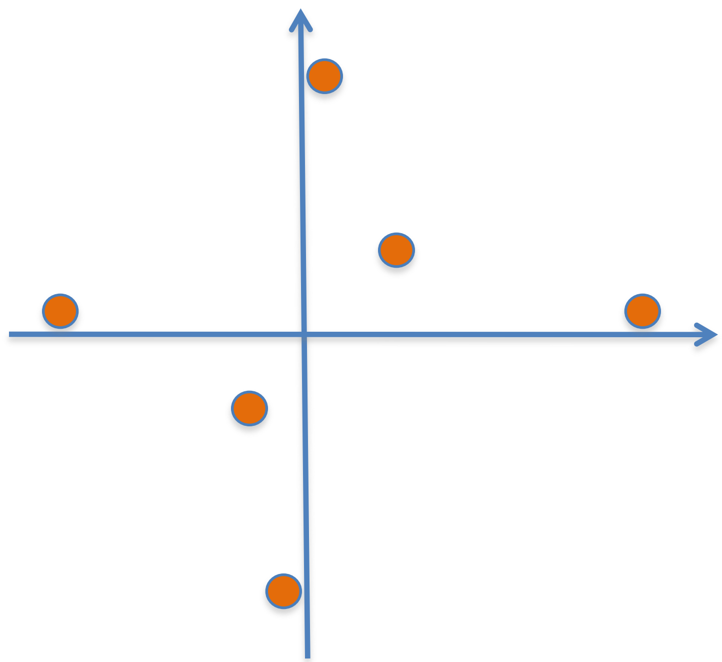
1-D problem in time $O(n \log n)$?

Sorting to rescue in 2-D?

Pick pairs of points closest in x co-ordinate

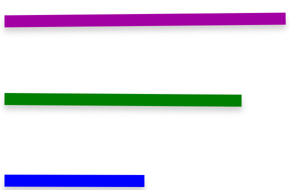
Pick pairs of points closest in y co-ordinate

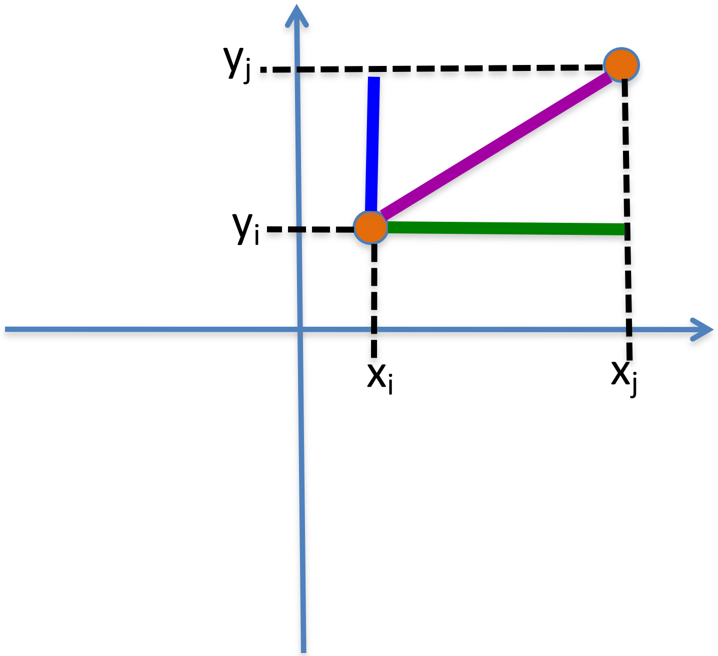
Choose the better of the two



A property of Euclidean distance

$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$



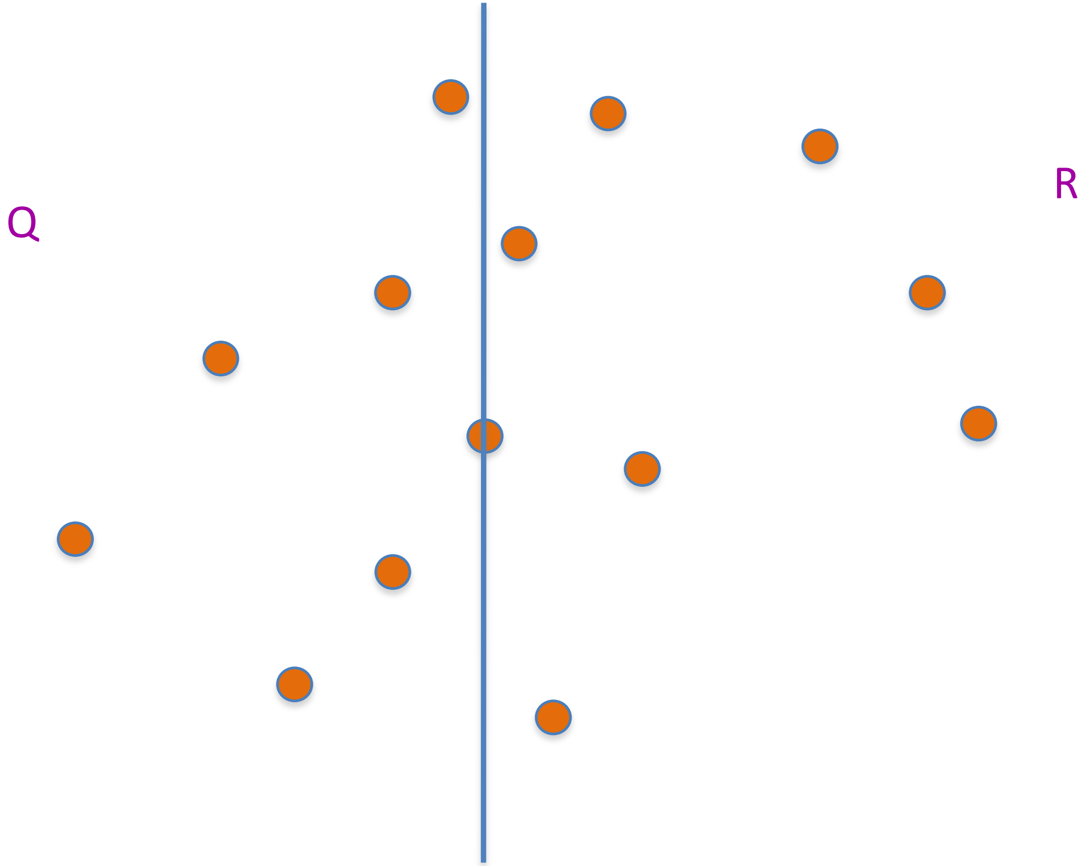


The distance is larger than the **x** or **y**-coord difference

Rest of Today’s agenda

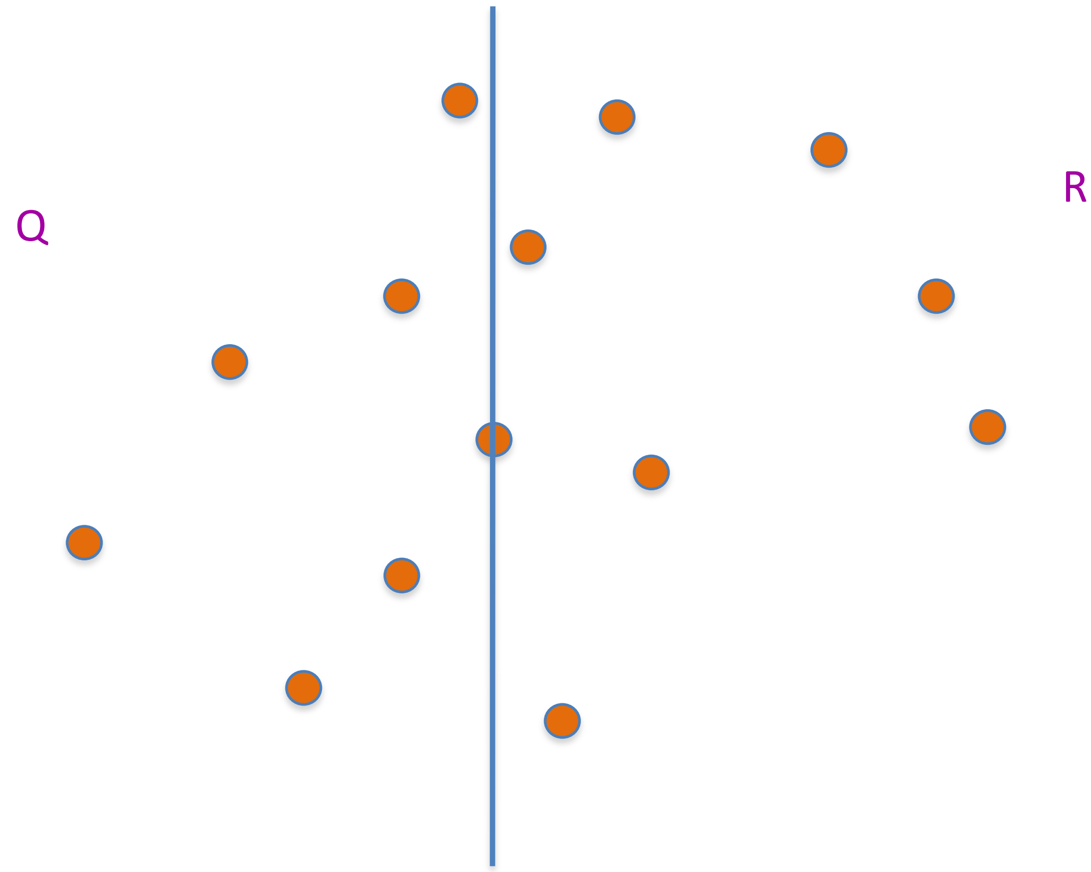
Divide and Conquer based algorithm

Dividing up P



First n/2 points according to the x-coord

Recursively find closest pairs



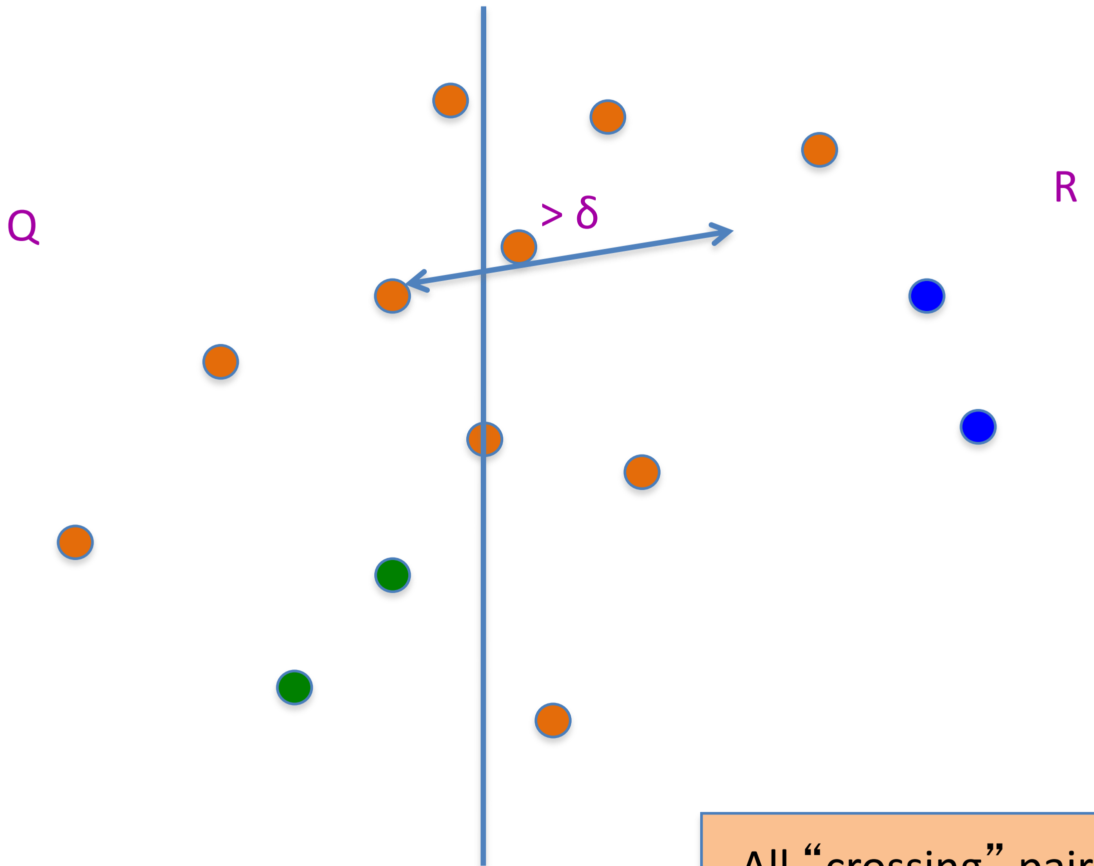
δ = min (**blue**, **green**)

An aside: maintain sorted lists

P_x and P_y are P sorted by x-coord and y-coord

Q_x, Q_y, R_x, R_y can be computed from P_x and P_y in $O(n)$ time

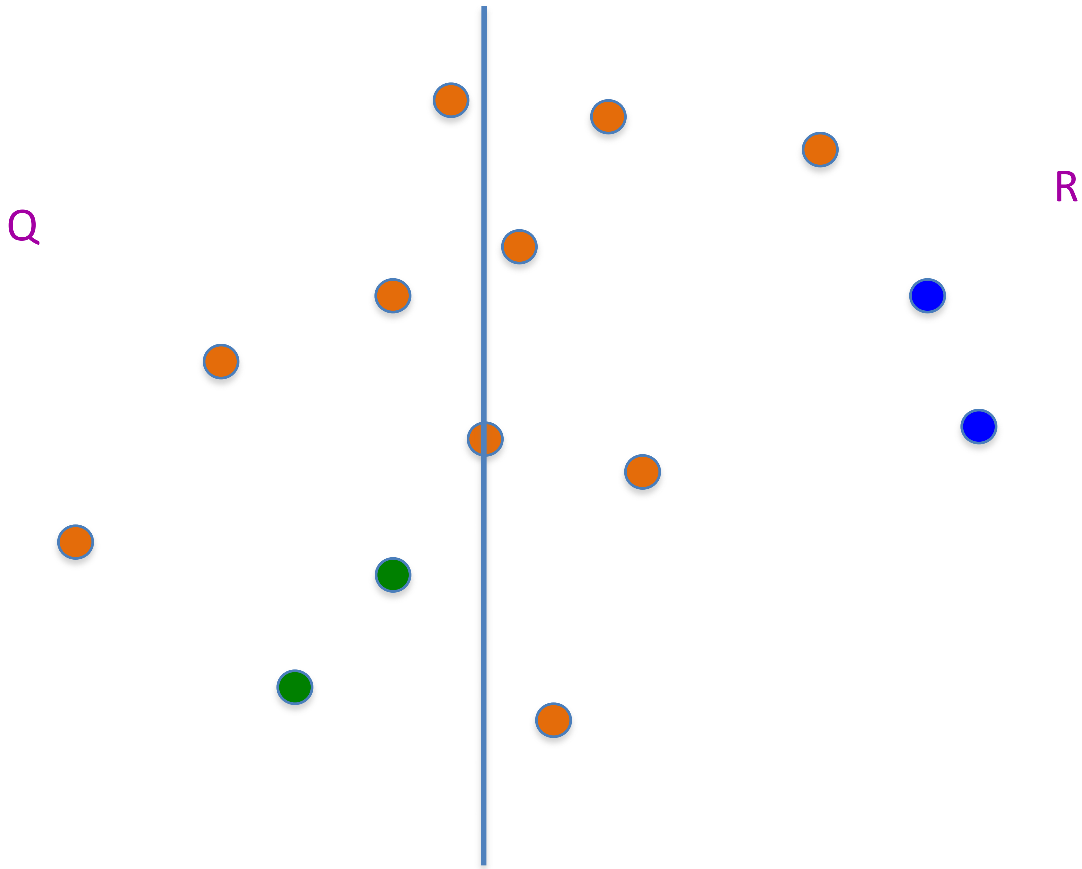
An easy case



All “crossing” pairs have distance > δ

δ = min (**blue**, **green**)

Life is not so easy though



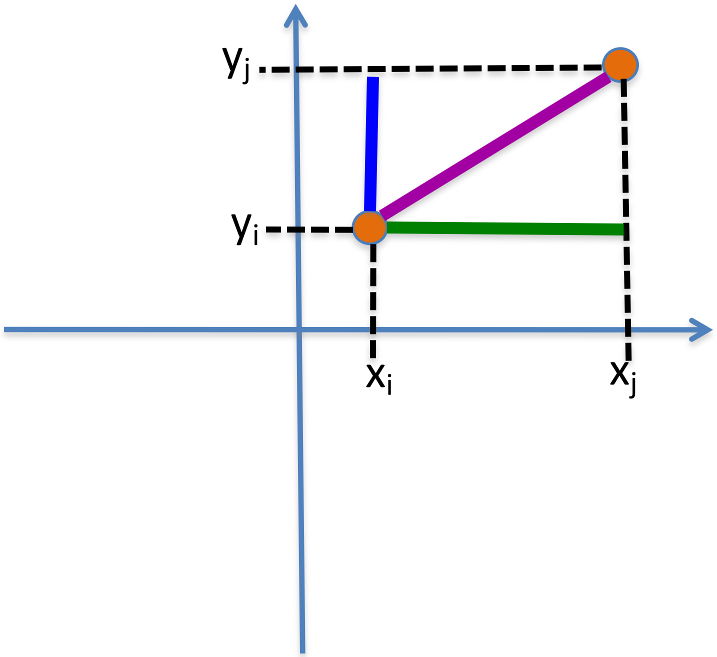
δ = min (**blue**, **green**)

Rest of Today’s agenda

Divide and Conquer based algorithm

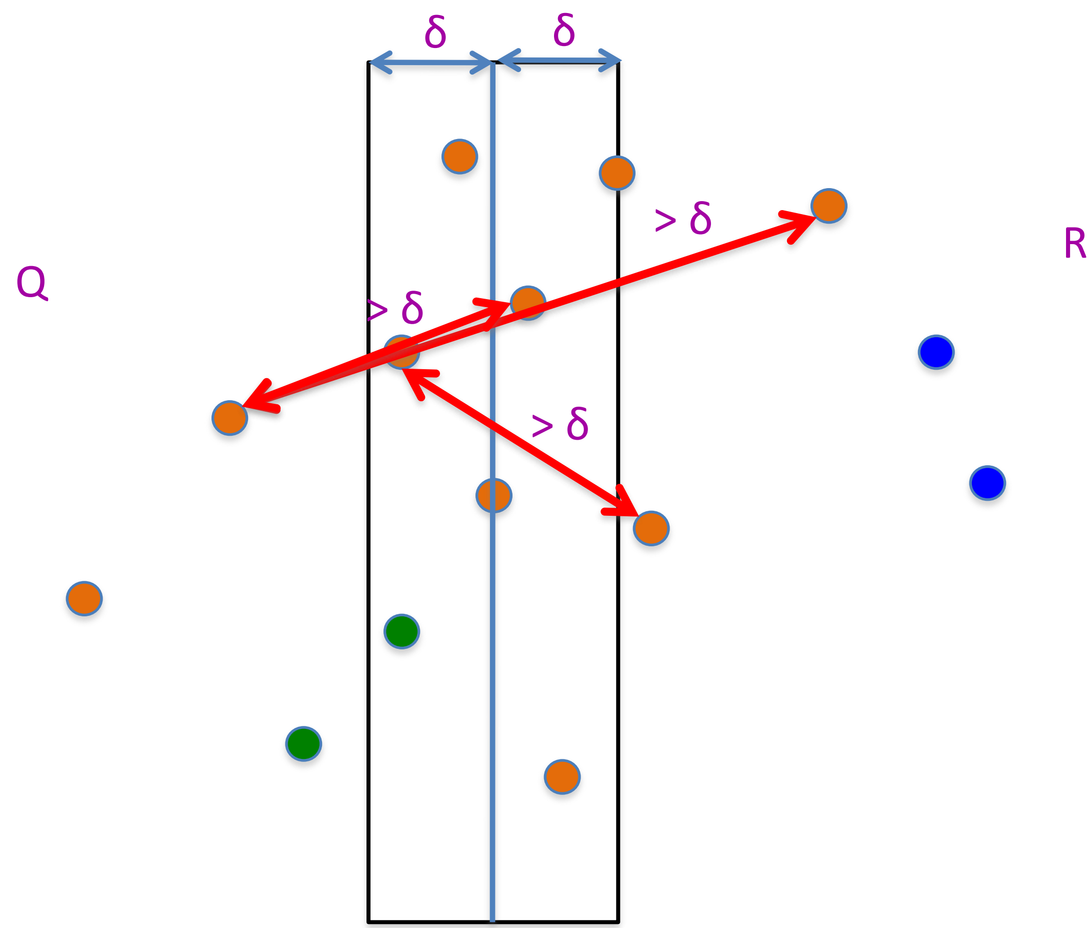
Euclid to the rescue (?)

$d(p_i,p_j) = ((x_i-x_j)^2+(y_i-y_j)^2)^{1/2}$



The distance is larger than the **x** or **y**-coord difference

Life is not so easy though



δ = min (**blue**, **green**)

All we have to do now

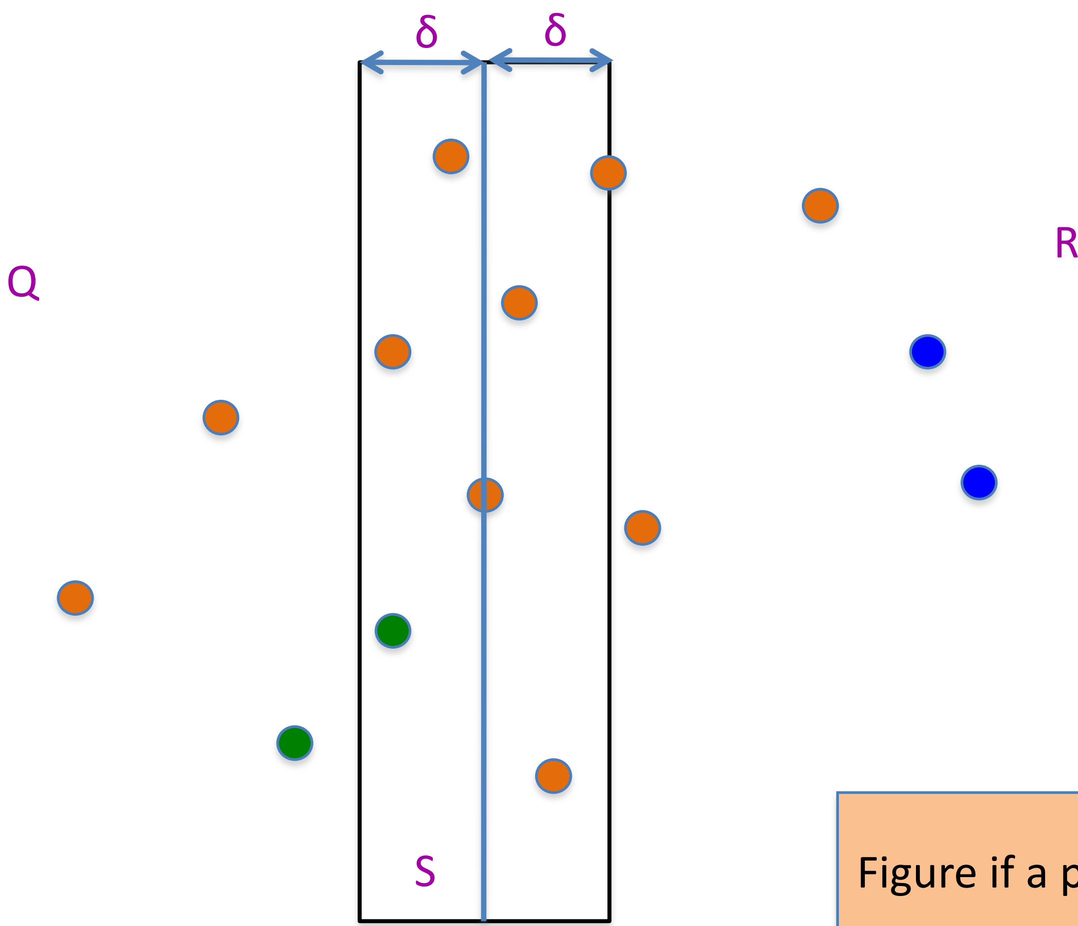


Figure if a pair in S has distance < δ

δ = min (**blue**, **green**)

The algorithm so far…

$O(n \log n) + T(n)$

Input: n 2-D points $P = \{p_1,\ldots,p_n\}$; $p_i=(x_i,y_i)$

Sort P to get P_x and P_y — $O(n \log n)$

Closest-Pair (P_x, P_y)

If $n < 4$ then find closest point by brute-force

Q is first half of P_x and R is the rest — $O(n)$

Compute Q_x, Q_y, R_x and R_y — $O(n)$

(q_0,q_1) = Closest-Pair (Q_x, Q_y)

(r_0,r_1) = Closest-Pair (R_x, R_y)

$\delta = \min(\, d(q_0,q_1), d(r_0,r_1)\,)$ — $O(n)$

S = points (x,y) in P s.t. $|x - x^*| < \delta$ — $O(n)$

return Closest-in-box (S, (q_0,q_1), (r_0,r_1)) — Assume can be done in $O(n)$

$T(< 4) = c$

$T(n) = 2T(n/2) + cn$

$O(n \log n)$ overall