Lecture 5

CSE 331

Sep 7, 2018

HW 1 posted

Homework 1

Due by **11:59pm, Thursday, September 13, 2018**.

Make sure you follow all the homework policies.

All submissions should be done via Autolab.

Post questions on Piazza!

Some Questions on Stable Matching

Sample Problem

The Problem

Decide whether the following statement is true or false:

In every Stable Marriage problem instance where a man m and woman w have each other as their least preferred partner, the following is true. There is no stable matching for the instance where (m, w) are matched.

If you state true then you will have to formally argue why the statement is correct. If you state false, then you have to give a counter-example.

Take note of the many(!) notes

! PDF only please

Autolab might not be able to display files in formats other than PDF (e.g. Word cannot be displayed). **If Autolab cannot display your file, then you will get a zero (0) on the entire question. Note that Autolab will NOT give an error message if you submit non-PDF file, so it is YOUR responsibility to make sure you submit in the correct format.** Also the file size has to be **at most 3MB**.

Grading Guidelines

We will follow the usual grading guidelines for non-programming questions. Here is a high level grading rubric specific to part `(a)` of this problem:

1. `Proof idea`: 10 points.

and here is the high level grading rubric for part `(b)`:

1. `Proof idea`: 17 points for a counterexample idea explaining the insight behind why you think the property does not holds.
2. `Proof details`: 18 points for a complete description of a counterexample and a complete proof for why the given counter example does not have any stable schedule.

! Note

If you do not have separated out proof idea and proof details for part `(b)`, you will get a zero(0) irrespective of the technical correctness of your solution..

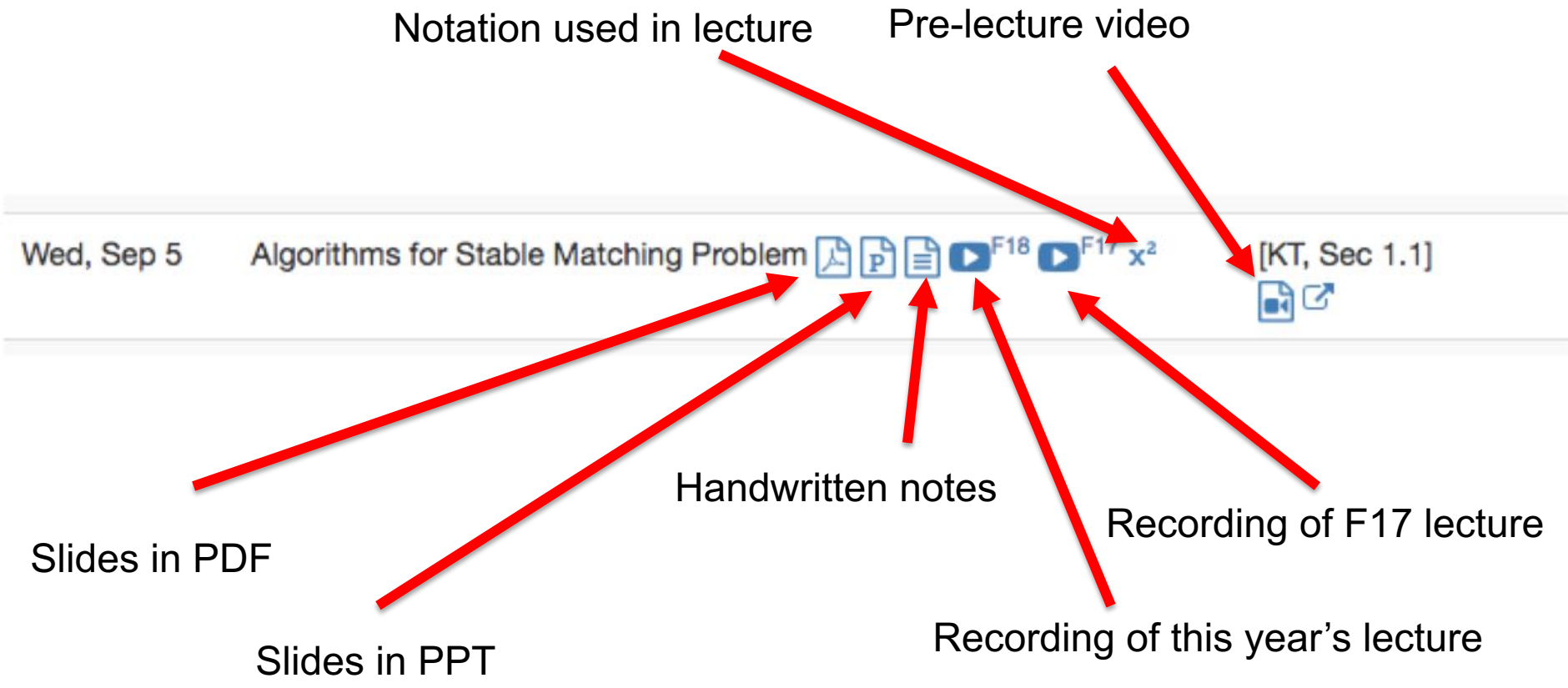
Templates

Download LaTeX template. Download Microsoft Word template.

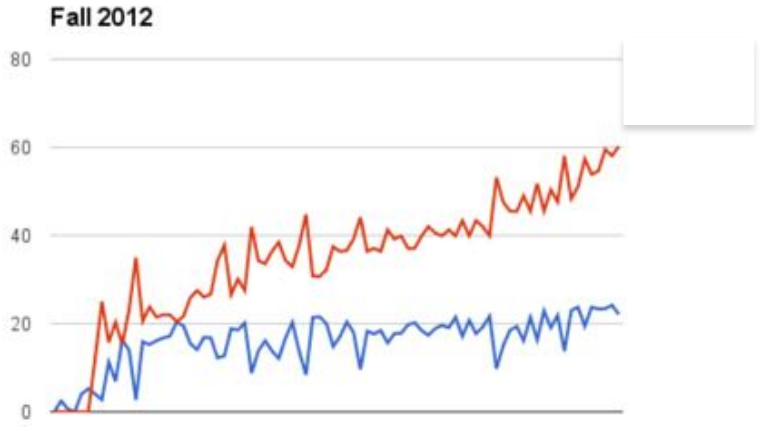
! Note

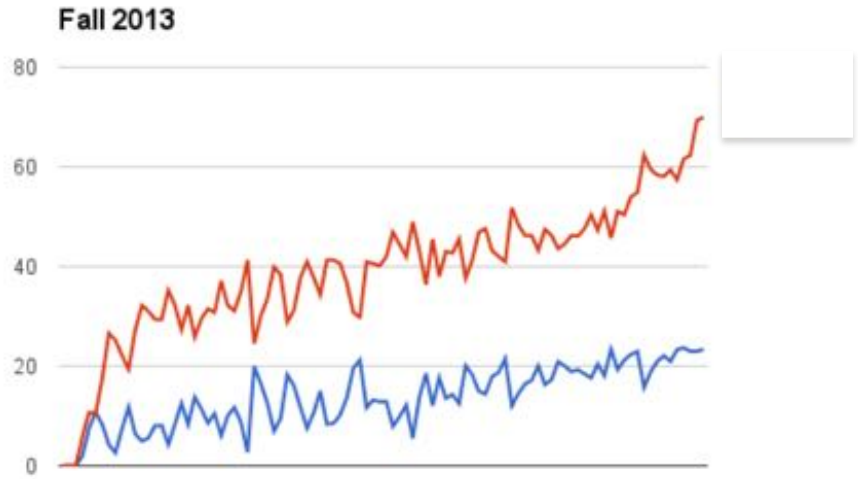
Your must explicitly list your sources and collaborators when you upload your submission to Autolab. Note that you can only used one of the five allowed sources. If you have used a source that is not allowed, please do **not** submit your homework. If you did not consult any source or did not collaborate with anyone just say None .

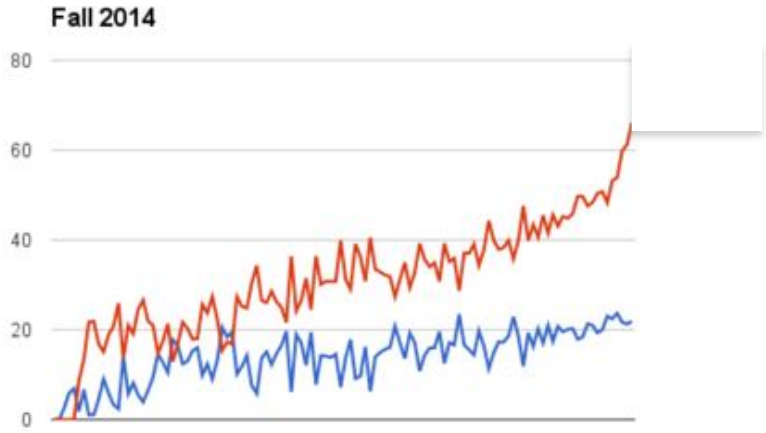
Various lecture related stuff



Can you guess the correlation?







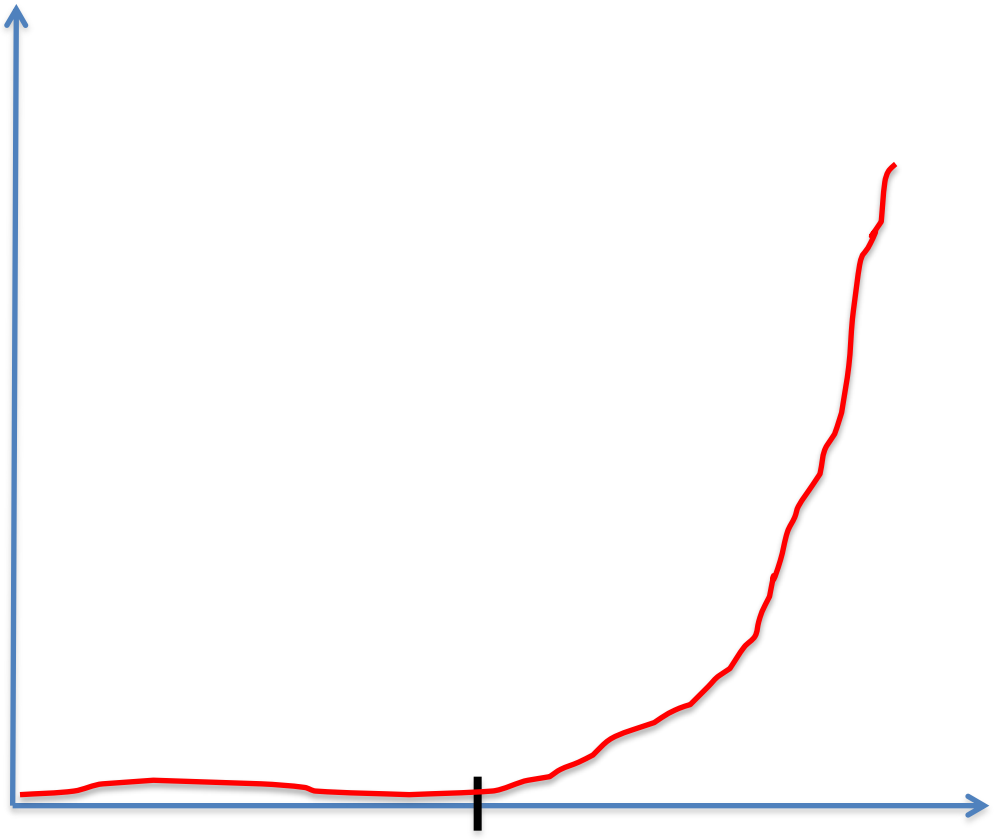
Another comment

Discomfort with proofs

I will not cover proof basics in class

Please read support pages and talk to us in person if you need help

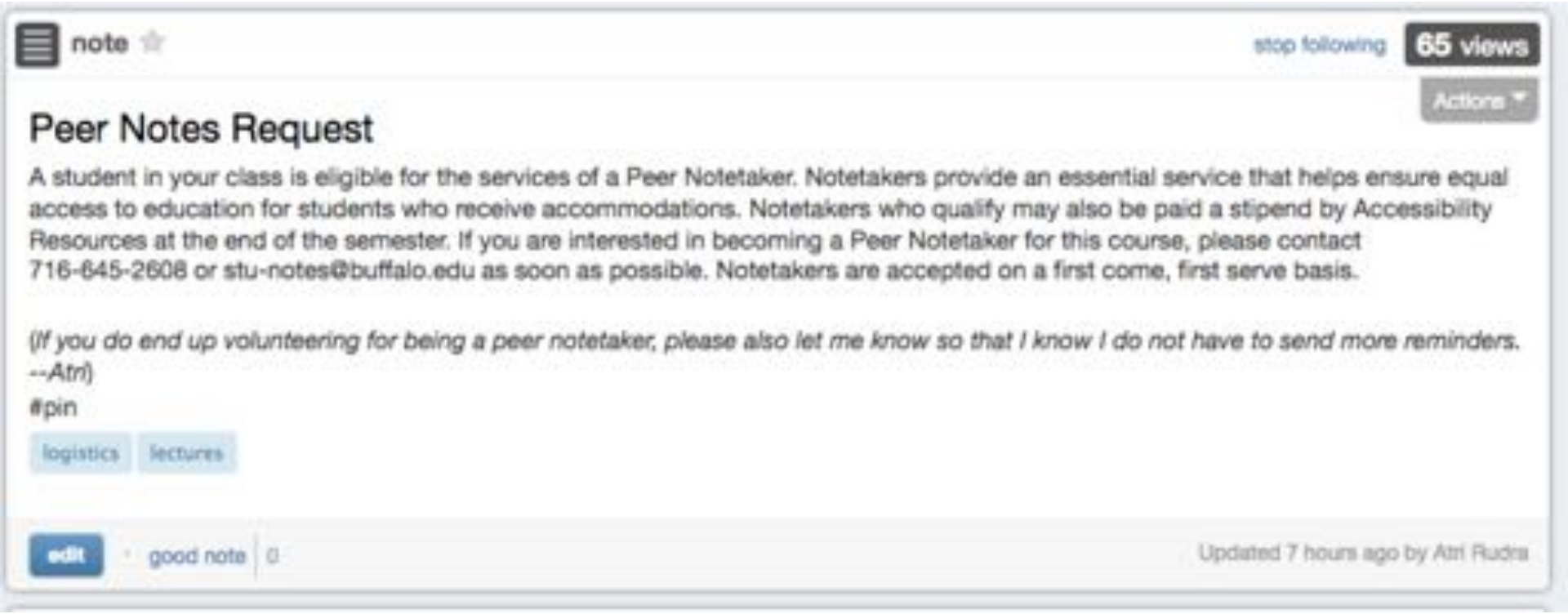
Lecture pace



We’re not mind readers

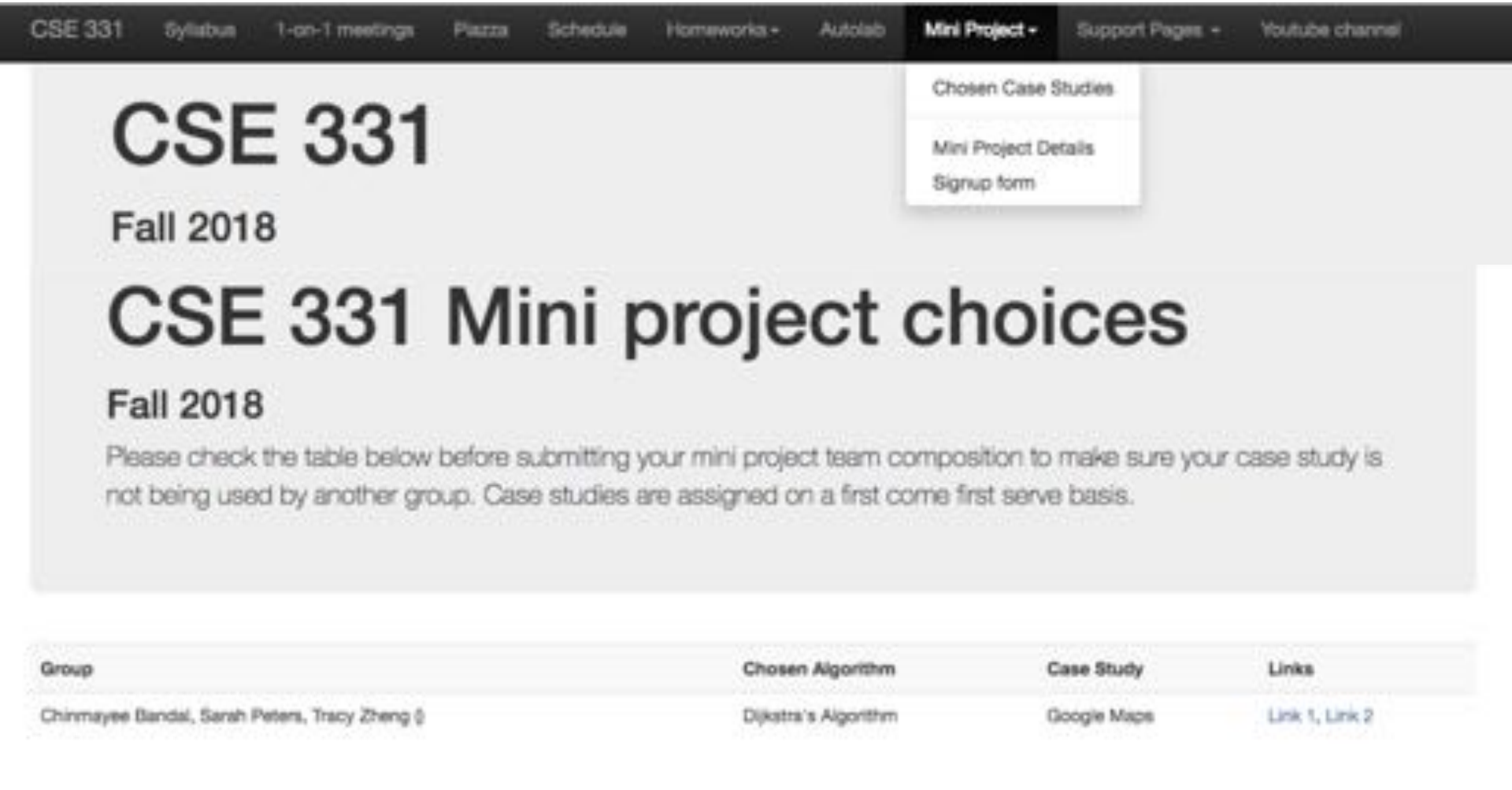
If you need it, ask for help

Peer Notetaker Request



Sign-up for mini projects

Deadline: Monday, Sep 24, 11:59pm



Questions/Comments?

Stable Marriage problem

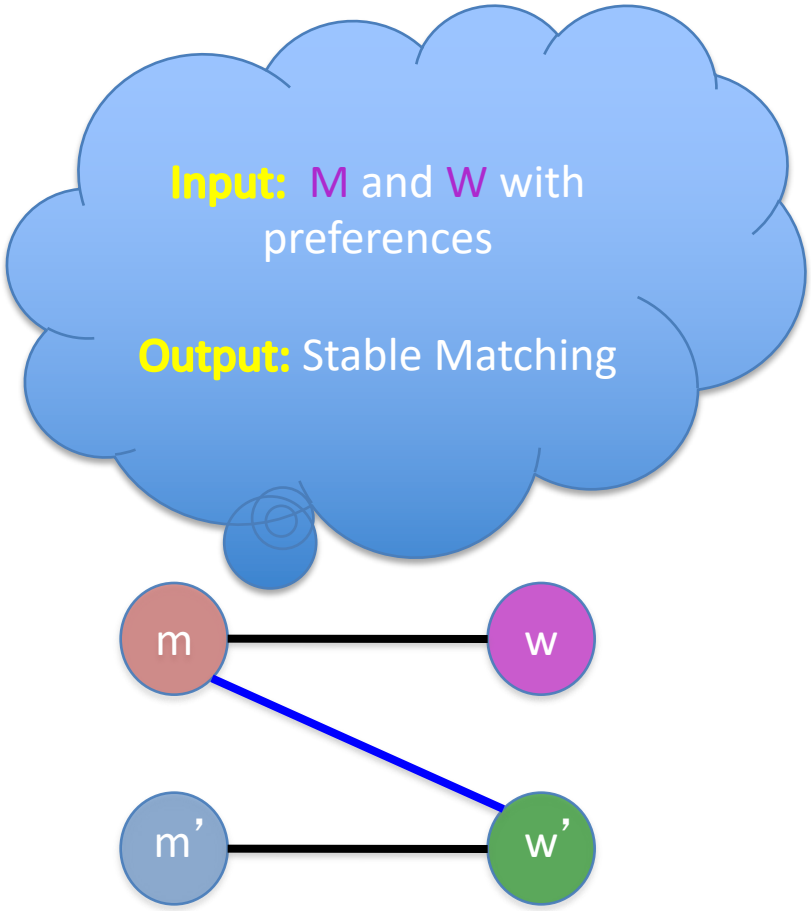
Set of men M and women W

Preferences (ranking of potential spouses)

Matching (no polyandry/gamy in M X W)

Perfect Matching (everyone gets married)

Instablity



Stable matching = perfect matching+ no instablity

Two Questions

Does a stable marriage always exist?

If one exists, how quickly can we compute one?

Today's lecture

Naïve algorithm

Gale-Shapley algorithm for Stable Marriage problem

The naïve algorithm

Incremental algorithm to produce all n! prefect matchings?

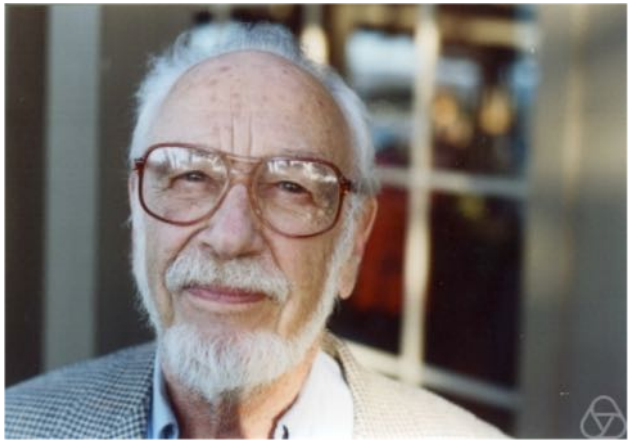
Go through all possible perfect matchings S

If S is a stable matching

then Stop

Else move to the next perfect matching

Gale-Shapley Algorithm



David Gale

Lloyd Shapley

$O(n^3)$ algorithm

Moral of the story…

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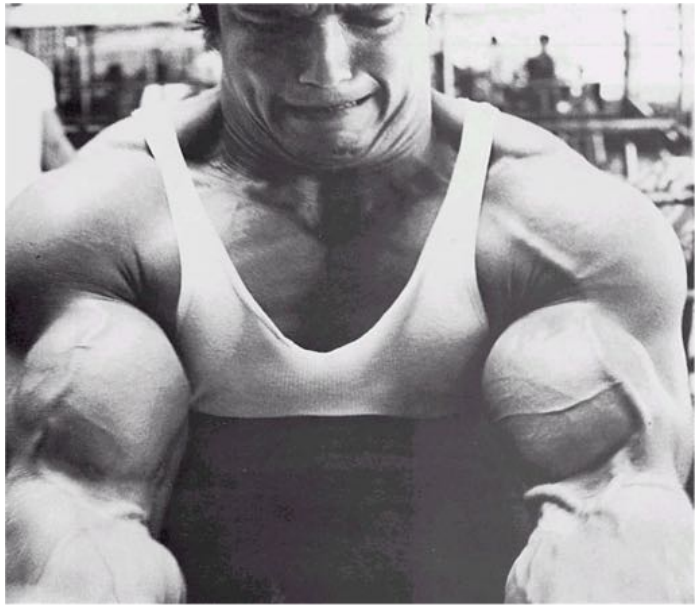


Questions/Comments?

Rest of today's agenda

Run of GS algorithm on an instance

Prove correctness of the GS algorithm

Gale-Shapley Algorithm

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

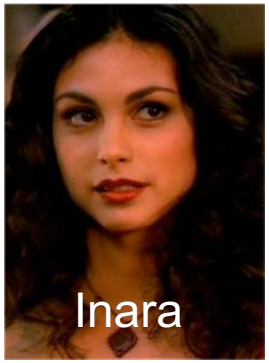
w remains free

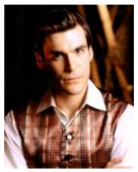
Else

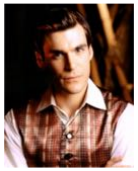
(m,w) get engaged and w' is free

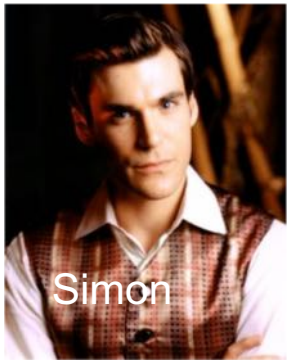
Output the engaged pairs as the final output

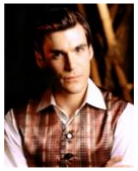
Preferences











GS algorithm: Firefly Edition

Observation 1

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

Once a man gets engaged, he remains engaged (to "better" women)

Output the engaged pairs as the final output

Observation 2

Intially all men and women are free

While there exists a free woman who can propose

Let w be such a woman and m be the best man she has not proposed to

w proposes to m

If m is free

(m,w) get engaged

Else (m,w') are engaged

If m prefers w' to w

w remains free

Else

(m,w) get engaged and w' is free

If w proposes to m after m', then she prefers m' to m

Output the set S of engaged pairs as the final output

Questions/Comments?

Why bother proving correctness?

Consider a variant where any free man **or** free woman can propose

Is this variant any different? Can you prove it?

GS’ does not output a stable marriage