Sep 19

$G = (V, E)$ — graph; V: set of vertices/nodes; E: set of edges

$E \subseteq V \times V$

Default notion: $n = |V|$, $m = |E|$

Def: G is undirected if $\forall\ u, w \in V$ $(u, w) \in E \iff (w, u) \in E$

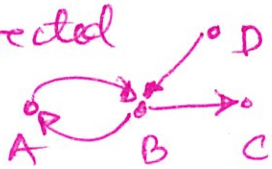
(•) New host graph (D)

(•) Airline map (U)

(•) Wikipedia pages (D)

(•) Internet graph (U)

o/w G is directed



By default: G will be undirected.

Claim: Every undirected graph is directed.

Pf idea:

Def: A path in $G = (V, E)$ is a sequence of vertices $u_1, \ldots, u_k$ s.t. $\forall\ i \in [k-1]$ $(u_i, u_{i+1}) \in E$

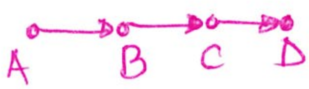
(i) u_i's need not be distinct (ii) G is directed or undirected.

D, C, B, A ✓

A, B, C, D ✓

A, B, C, ~~D~~B ✓

A, C, D ✗



D, C, B, A ✗

A, B, C, D ✓

A, B, C, B ✗

A, C, D ✗

Def: A simple path does not have any repeated vertices

By default: All paths are simple.

Def: Length of a path is # of edges in it

Q: What is the max length of a simple path?

A: $n-1$

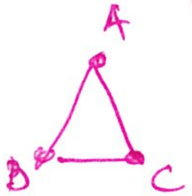
Pf idea: A path of length $\geq n$ has $\geq n+1$ nodes $(u_1, \ldots, u_{n+1})$. $n+1$ labels but n vertices. By pigeon hole principle $\exists\ i \neq j$ s.t. $u_i = u_j$ $\Rightarrow$ not a simple path.

Def: A cycle $u_1, \dots, u_k$ is a path s.t
(i) $u_1, \dots, u_{k-1}$ are distinct (ii) $u_1 = u_k$ (iii) If G is directed $k \geq$

✓ u, w, u  i u, u

If G is undirected $k \geq 4$

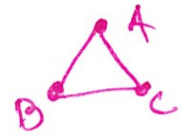
A, C, B, A ✓



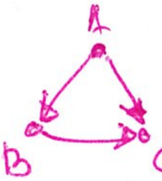
A, B, C, A ✓
"triangle"

A, B, C, A ✓
A, C, B, A X

✗ u, w, u



A, B, C, A ✓



A, B, C, A X
A, C, B, A X

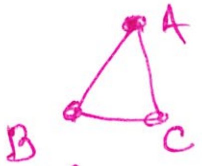
Def: u & w are connected (undirected) if $\exists$ a u–w path

________ strongly connected (directed) if $\exists$ u–w path
w–u path.

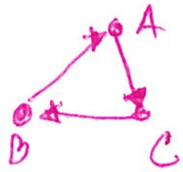
A, C connected? ✓

strongly connected?
X

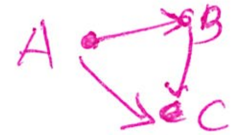
Def: A (directed) graph G is (strongly) ~~con~~ connected if $\forall$ $u \neq w \in V$, u & w are (strongly) connected.



C? ✓



SC? ✓



SC? X.