Sep 24

PROPOSITION: Let T be a BFS tree for $G = (V, E)$

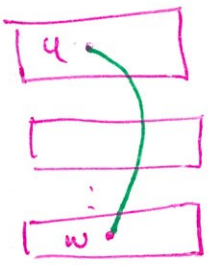
If $(u, w) \in E$ s.t $u \in L_i$, $w \in L_j$

$\Rightarrow |i - j| \leq 1 \iff i \in \{j-1, j, j+1\}$

Pf idea: WLOG (without loss of generality) assume $i \leq j$ (If not, switch i & j)

for contradiction assume $|i - j| > 1 \Rightarrow j > i+1$

$\iff j \geq i+2$



Consider the situation when BFS was creating L_{i+1}

$\rightarrow u \in L_i$; $w \notin L_0, \dots, L_i$

$\rightarrow (u, w) \in E$

$\Rightarrow w$ satisfies the condition of being added to L_{i+1}

$\Rightarrow$ contradicts $w \in L_j$ where $j > i+1$.