Sep 26

Explore (s)

0. $R = \{s\}$

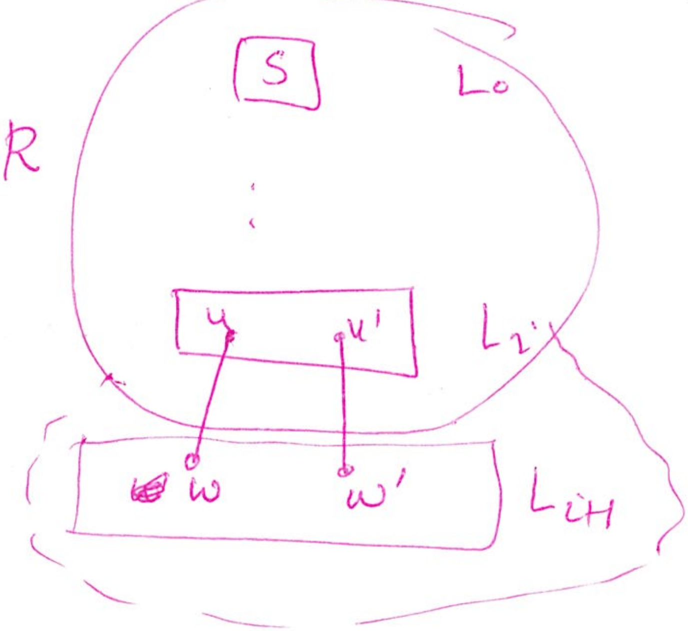
1. While $(u,w) \in E$ s.t. $u \in R$ and $w \notin R$

   Add w to R

2. Return $R^* = R$

such that

Consider the time when L_{i+1} is being created

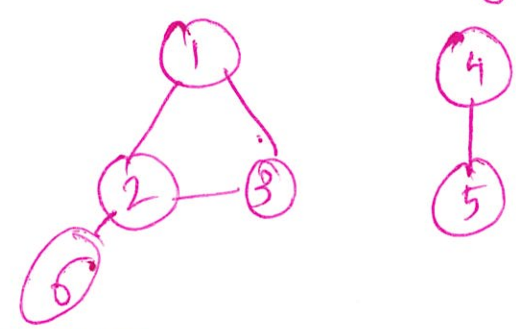


Def: Set of all vertices connected to s is called its connected component. Denote it by $CC(s)$

THEOREM: $C(s) = R^*$

General trick to show two sets $A = B$

$\iff$ $A \subseteq B$ and $B \subseteq A$.



$CC(1) = \{1,2,3,6\}$

$CC(4) = \{4,5\}$

LEMMA 1: $R^* \subseteq CC(s)$

LEMMA 2: $CC(s) \subseteq R^*$

LEMMAS 1+2 $\Rightarrow$ THM.

Ex: Prove by induction

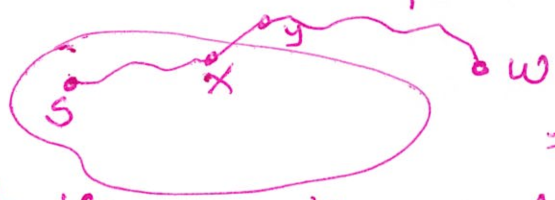
Cor: BFS is correct.

Pf idea of Lemma 2: By contradiction.

Assume $CC(s) \not\subseteq R^*$ $\Rightarrow$ $\exists\ w \in CC(s)$ BUT $w \notin R^*$

$\Leftrightarrow$ $\exists$ s-w path P but $w \notin R^*$

R^*



Since P starts inside of R^* & ends up outside of R^*

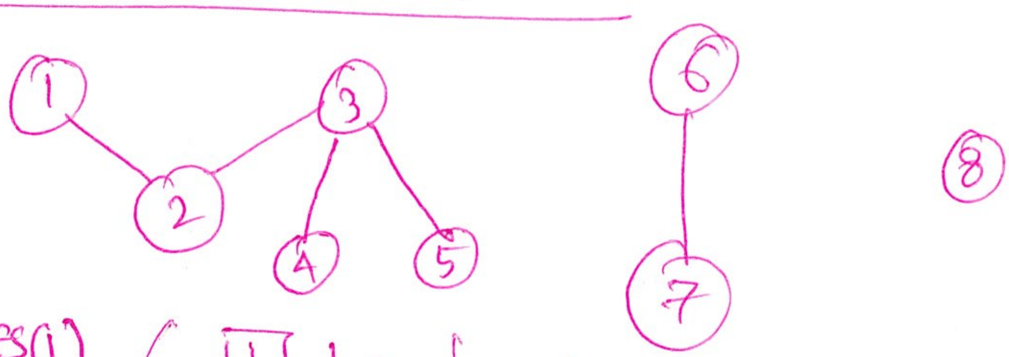
$\Rightarrow \exists\ (x, y) \in P$ s.t. $x \in R^*$, $y \notin R^*$

But then y is a candidate to be added to R

$\Rightarrow$ Algo has not terminated

$\Rightarrow$ contradicts the fact that algo has output R^* (and hence terminated) ∎

Compute all CC of G:



BFS(1)

$CC(1) = \{1, 2, 3, 4, 5\}$

$\boxed{1}$ L_0

$\boxed{2}$ L_1

$\boxed{3}$ L_2

$\boxed{4\ \ 5}$ L_3

BFS(6)

$\boxed{6}$ L_0

$\boxed{7}$ L_1

$CC(6) = \{6, 7\}$

BFS(8)

$\boxed{8}$ L_0

$CC(8) = \{8\}$

Note: Used DFS/ Explore instead of BFS.