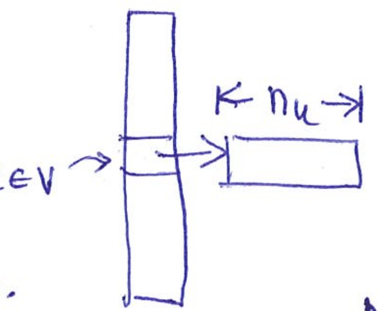
Sep 28

Adjacency list format

n_u = # neighbors of u $= |\{w \mid (u,w) \in E\}|$
(= degree of u)

Recall $2m = \sum_{u \in V} n_u$



pointers $= |V| = n$
Sum of list sizes $= \sum_{u \in V} n_u = 2m.$
$\Rightarrow$ overall space $= n + 2m = \Theta(n+m)$

$O(n) \swarrow \quad \searrow O(n^2)$

Note: $0 \le m \le \binom{n}{2} \le O(n^2)$

BFS (G, s) // G is in adj. list format

0. $CC[s] = T$ and $CC[u] = F \quad \forall\, u \ne s \in V$
1. $i = 0$
2. $L_0 = \{s\}$ $\Big\}\ O(n)$
3. While $L_i \ne \phi$ $\}$ T_1: # times this loop is run
   - 3.1 $L_{i+1} = \phi \leftarrow O(1)$
   - 3.2 For all $u \in L_i$
     - For $(u,w) \in E \leftarrow n_u \le n$
       - If $CC[w] == F$
       - $CC[w] = T$
       - Add w to L_{i+1} $\Big\}\ O(1)$
   - 3.3 $i{+}{+} \leftarrow O(1)$
4. Return $CC \leftarrow O(n)$

T_{12}: # times control get here

T_{123}: # times this line gets executed

$$\boxed{\begin{aligned} T_1 &\le T_{123} \\ T_{12} &\le T_{123} \end{aligned}}$$

Total runtime: $O(n) + \underset{\substack{\uparrow \\ \#\ 3.1\ \text{is executed}}}{T_1} \cdot O(1) + T_{123} \cdot O(1) + \underset{\substack{\uparrow \\ 3.3\ \text{is executed}}}{T_{12}} \cdot O(1)$

$\le O(n) + T_{123} \cdot O(1) + T_{123} \cdot O(1) + T_{123} \cdot O(1) = O(n) + T_{123}$

Goal: Bound T_{123}

Analysis 1: Claim: $T_{123} \leq O(n^3)$ $\Rightarrow$ overall: $O(n) + O(n^3) = O(n^3)$

Each vertex u is in at most one L_i

$\Rightarrow T_1 \leq n$

time 2nd for loop runs $\leq n$, 3rd for loop $\leq n$

$\Rightarrow T_{123} \leq n \cdot n \cdot n = n^3$

Analysis 2. Claim 2: $T_{123} \leq n^2$ $\Rightarrow$ overall: $O(n+n^2) = O(n^2)$

Claim 3: $T_{12} \leq n$

$\Rightarrow T_{123} \leq T_{12} \cdot n = n^2$.

Analysis: Claim 4: T_{123} is $O(m)$ $\Rightarrow$ overall: $O(n) + O(m)$

$= O(n+m)$

BFS is linear time $\Leftarrow$ input size $= \Theta(n+m) = N$

$$T_{123} = \underbrace{\sum_{i=1}^{T_1} \sum_{u \in L_i}}_{\text{each } u \text{ in} \leq \text{one } L_i} n_u$$

$$\leq \sum_{u \in V} n_u = 2m \quad \blacksquare$$