Oct 8

THM1: S^* is an optimal solution
↳ among all valid schedules with max # intervals

Ex1: Algo terminates

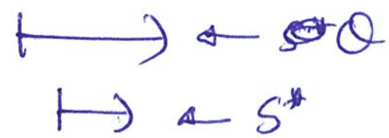
Ex2: S^* is a valid schedule

Pf of correctness of greedy algo → Greedy stays ahead (next) ; → Exchange argument (later)

Let O be an optimal solution

Ex: Convince yourself that such an optimal solution $\exists$.

Idea 1: $S^* = O$ } problem: have multiple optimal solutions



Idea 2: Show $|S^*| = |O|$

THM 2: $|S^*| = |O|$

Notation: $S^* = \{i_1, \ldots, i_k\}$ $\quad O = \{j_1, \ldots, j_m\}$

$f(i_1) \le f(i_2) \le \cdots \le f(i_k)$ $\quad f(j_1) \le \cdots \le f(j_m)$

THM2': $k = m$

Claim: $k \le m$

Lemma 1 ("Greedy stays ahead") $\forall\ 1 \le l \le k$

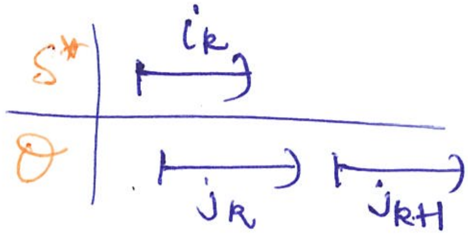
$f(i_l) \le f(j_l)$.

Pf idea for Thm 2': (Assume Lemma 1.)

Pf by contradiction: Assume $k \neq m$

$\xrightarrow{\text{Claim}}$ $k < m \iff m \geq k+1$

$\Rightarrow j_{k+1} \in \mathcal{O}$



By Lemma 1, $f(i_k) \leq f(j_k)$

Consider the case when Greedy adds i_k to S

$\Rightarrow j_{k+1} \in R \Rightarrow R \neq \emptyset \Rightarrow$ Greedy did not terminate

as j_{k+1} does not conflict with anything is S^* $\Rightarrow$ contradiction ■