Oct 10

Lemma 1: $\forall\ 1 \le \ell \le k$ $\quad$ $S^* = \{i_1, \dots, i_k\}$

$f(i_\ell) \le f(j_\ell)$ $\quad$ $\mathcal{O} = \{j_1, \dots, j_m\}$

$k \le m$

Last time: Lemma 1 $\Rightarrow k = m$ (i.e. Greedy is correct)

Pf idea of Lemma 1: By induction on ℓ.

Base case: $\ell = 1$ $\quad f(i_1) \le f(j_1)$ ← By algo def. $f(i_1)$ has smallest finish time.

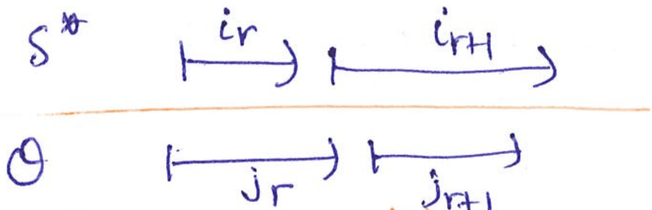
I.H.: $f(i_\ell) \le f(j_\ell)$ $\forall\ 1 \le \ell \le r$

$r \ge 1$

I.S: To show $f(i_{r+1}) \le f(j_{r+1})$

For the sake of contradiction assume $f(i_{r+1}) > f(j_{r+1})$

(•) Consider the situation when i_r is added to S



$f(j_r) \ge f(i_r)$ by I.H.

(*) $i_{r+1}, j_{r+1} \in R$

Ex: Argue that j_{r+1} does not conflict $i_1 \dots i_r$

But $f(j_{r+1}) < f(i_{r+1})$

$\Rightarrow$ greedy could not have chosen i_{r+1} next $\Rightarrow$ contradiction.

Recall: $f(1) \le \dots f(n)$

Greedy algo:

0) $R = [n]$ ($\stackrel{\text{def}}{=} \{1, \dots, n\}$) ← $O(n)$

1) $S = \phi$ ← $O(1)$

2) While $R \ne \phi$ ← $\le n$

$O(n)$ { (2.1) Pick $i \in R$ with the smallest index ← $O(n)$

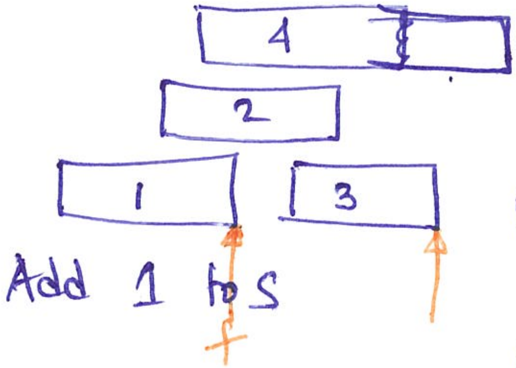
(2.2) Add i to S ← $O(n)$

(2.3) Remove all $j \in R$ that conflict with i ← $O(n)$ }

throws away atleast i

3) Return $S^* = S$ ← $O(n)$

$O(n) + O(1) + n \cdot O(n) + O(n) = O(n^2)$



$f(1) < f(2) < f(3) < f(4)$

Old version:
- → delete 2 and 4
- → Add 3 to S

New version:

(A) $f = f(1)$

(B) Skip over 2 as $s(2) < f(1)$

(C) Add 3 to S as $s(3) \geq f \; (= f(1))$

(D) $f = f(3)$

(E) Skip over 4 as $s(4) < f \; (= f(3))$