Oct 22

Let S be schedule of greedy algo

Let O be an optimal solution

Recall: $L(S)$ is the max lateness of S $\quad \ell_i = \max(0, f(i) - d_i)$ $\quad L(S) = \max_i \ell_i$

THM 1: $L(S) = L(O)$

Def: Idle time of a schedule = max gap b/w any 2 consecutively scheduled jobs

Obs 1: S has 0 idle time

Obs 2: Can assume O has 0 idle time (If not "squish" jobs)

Def: Let S be a schedule. A pair of jobs (i, j) is an inversion if (1) $d_i > d_j$ AND (2) i is scheduled before j [$f(i) \le s(j)$]

Obs 3: S has 0 inversions

Lemma 1: If S_1 and S_2 have both 0 idle time and 0 # of inversions $\Rightarrow L(S_1) = L(S_2)$

Lemma 2: S has 0 idle time and 0 #inversions ← obs 1+3

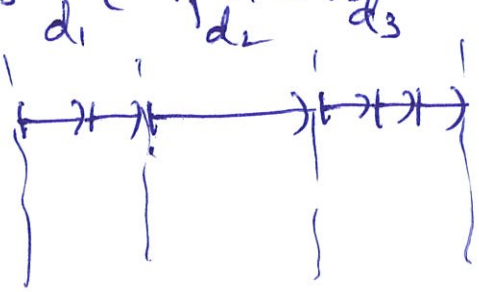
Lemma 3: There exists an optimal schedule, O, with 0 idle time & 0 # inversions.

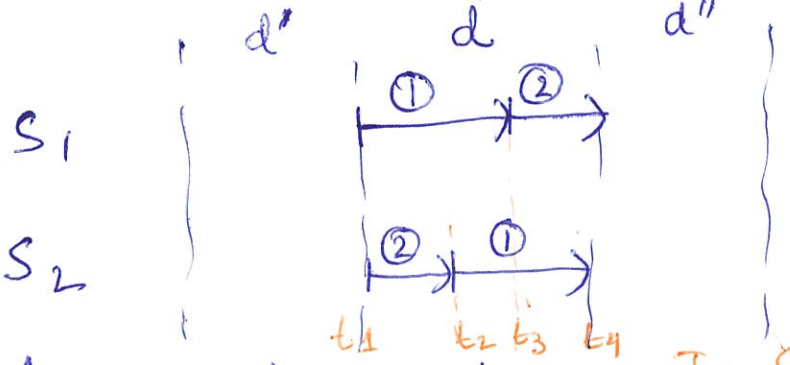
Lemmas 1+2+3 $\Rightarrow L(S) = L(O) \Rightarrow$ THM 1.

(Pf idea of Lemma 1)

Claim: For any schedule with 0 idle ~~idle~~ time and 0 # inversions, all jobs i s.t. $d_i = d$ are scheduled right next to each other (Pf: later)

$\Rightarrow$ only difference between S_1 & S_2 is how jobs with same deadline are scheduled





$d' < d < d''$

& 0 # inversions.

Assume Claim is true

In S_1: $\ell_1 = \max(0, t_3 - 1 - d)$, $\ell_2 = \max(0, t_4 - 1 - d)$

$\max(\ell_1, \ell_2) = \max(0, t_4 - 1 - d)$

In S_2: $\ell_1 = \max(0, t_4 - 1 - d)$, $\ell_2 = \max(0, t_2 - 1 - d)$

$\max(\ell_1, \ell_2) = \max(0, t_4 - 1 - d)$

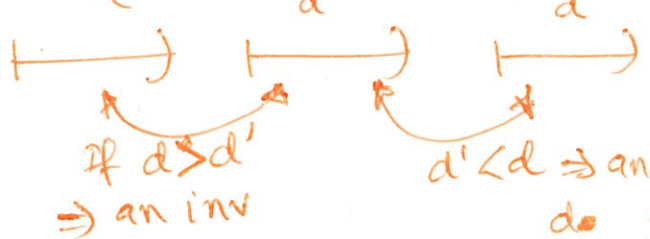
$\Rightarrow$ max lateness among all jobs with same deadline d does not change

$\Rightarrow$ $L(S_1) = L(S_2)$ ■ ← (except for Claim)

(since d was arbitrary)

Pf idea of Claim: ① 0 # inversions $\Rightarrow$ any 2 jobs with same deadline are scheduled "together"

If not



If $d > d'$ $\Rightarrow$ an inv

$d' < d \Rightarrow$ an inv $\Rightarrow$ contradiction

$d_0 < d' < d''$ ...

0 idle time $\Rightarrow$ no gap