Oct 26

Shortest Path Problem

Input: Directed graph $G = (V, E)$

$s \in V$

"length" $\rightarrow l_e \geq 0 \quad \forall e \in E$

integer

Output: $\forall t \in V$, output a shorstest s-t path (shortest length)

$$l(P) = \sum_{e \in P} l_e$$

{ Simpler version: only output $d(t)$ $\forall t \in V$

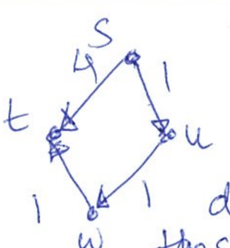
lengen of any shortest s-t path

Special case: $l_e = 1$ $\forall e \in E$. By HW 4 Q1 [run BFS on s & layer # of t is $d(t)$]

General case: $l_e \geq 0$ $e \in E$ **Idea:** Reduce this to $l_e = 1 \ \forall e \in E$

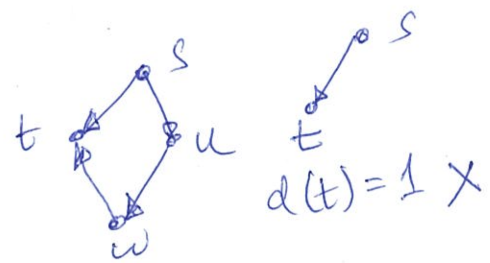
Idea 1: Ignore l_e (i.e. just assume $l_e = 1$ $\forall e \in E$ & run HW 4 Q1 algo)

Doesn't work:



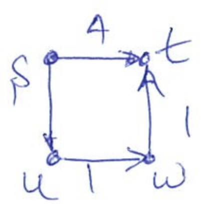
$d(t) = 3$
the shortest path s, u, w, t

Run
ignore l_e

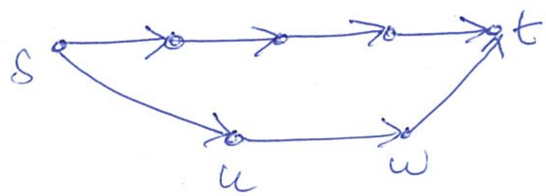


$d(t) = 1$ X

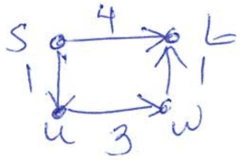
Idea 2: Replace each edge $e \in E$ by a path ~~of length~~ with l_e edges

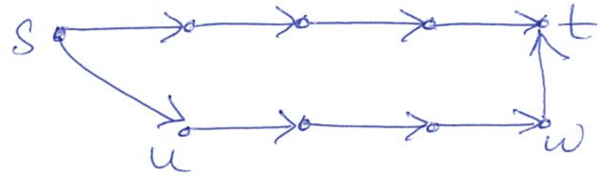


Works.



$l_e = 1$ $\forall e \in E$





(Each edge $\rightarrow$ path with l_e edges, all new nodes are "fresh")

Claim 1: shortest s-t path in G^ℓ ($l_e \geq 1$) $\Longleftrightarrow$ shortest s-t path in G' ($l_e = 1$)

Pf: Ex

$\Rightarrow$ Run HW4 Q1 algo

Correcbess: Claim 1 + HW4Q1

Runtime Analysis: $l_{max} = \max_{e \in E} l_e$

n' & m' be # edges in G'

$n', m' = O(l_{max}(n+m))$

Runtime $= O(n'+m') = O(l_{max}(n+m))$

Input size: each length l_e $O(\log l_{max})$ bits

input size $= O(\log l_{max}(n+m))$ ISSUE!

$O(n+m)$ registers

$l_{max} = n^{100}$

i/p size $= O((n+m)\log n)$

runtime $= O(n^{100}(n+m))$

Idea: Add in binary

Reminder: RAM model: basic ops on $O(\log n)$ bit #s are $O(1)$ time

$O(1)$ registers.

ASSUME: l_{max} is $n^{O(1)}$ $\Rightarrow$ $O(1)$ registers of each l_e

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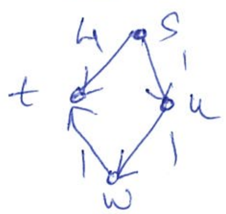
Toward's Dijkstra's algo

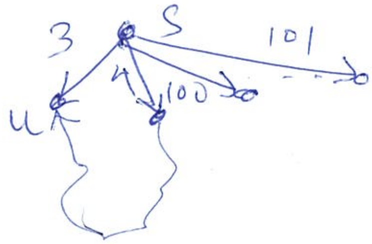


(s,t) is not a shortest s-t path

(s,u) IS ——— s-u path.

Claim: Pick u s.t. $l_{(s,u)}$ is min

$\Rightarrow (s,u)$ is a shortest s-u path.



Consider any other s-u path P

$l(P) = 4 + \underbrace{\text{stuff}}_{\geq 0 \text{ as } l_e \geq 0 \ \forall e \in E}$

$\geq 4 > 3 = l(s,u)$