

Nov 5

Perturbation trick

(e_1, \dots, e_m)

Assume: All c_e 's are integers

Idea: Add to the i th edge cost an "extra" $\frac{i}{2mn}$

$$c'_{e_i} = c_{e_i} + \frac{i}{2mn} \quad 1 \leq i \leq m$$

Ex: All k edge costs are distinct

Q: How much can the MST cost change.

In ~~the worst~~ any case the max perturbation

$$\leq \frac{m}{2mn} + \frac{m-1}{2mn} + \frac{m-2}{2mn} + \dots + \frac{m-(n-2)}{2mn} \text{ distinct cost}$$

$$< (n-1) \frac{m}{2mn} = \frac{n-1}{2n} < \frac{1}{2} \Rightarrow \text{cannot confuse any two spanning tree as}$$

~~distinct~~ distinct spanning tree costs differ ≥ 1 (as all c_e are integers).