Nov 16

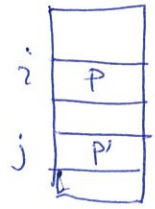
KICKASS PROPERTY LEMMA

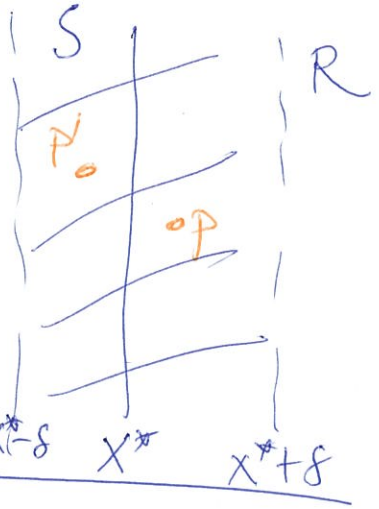
For every $p \neq p' \in S$ s.t $d(p,p') < \delta$ Q

s.t $S_y[i] = p$

$S_y[j] = p'$

$\Rightarrow |i-j| \leq 15$



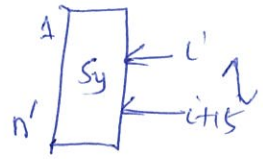


NOTE: (i) Can make "15" to be "9" (Ex.)
Can be as small as 7

(ii) Can compute S_y from P_y in $O(n)$ time

Q: How does the Kickass property lemma imply $O(n)$ Closest-in-Box algo?

for $i = 1 \ldots n'-1$



Let (p_i, p'_i) be closest pair of pts

$(S_y[i], S_y[i+1]), (S_y[i], S_y[i+2]), \ldots,$

$(S_y[i], S_y[i+15])$ $\min(i+15, n')$

Let (p,p') be closest pair of pts in

$(p_1, p'_1), \ldots, (p_{n'-1}, p'_{n'-1})$

If $d(p,p') < \delta$

return (p,p')

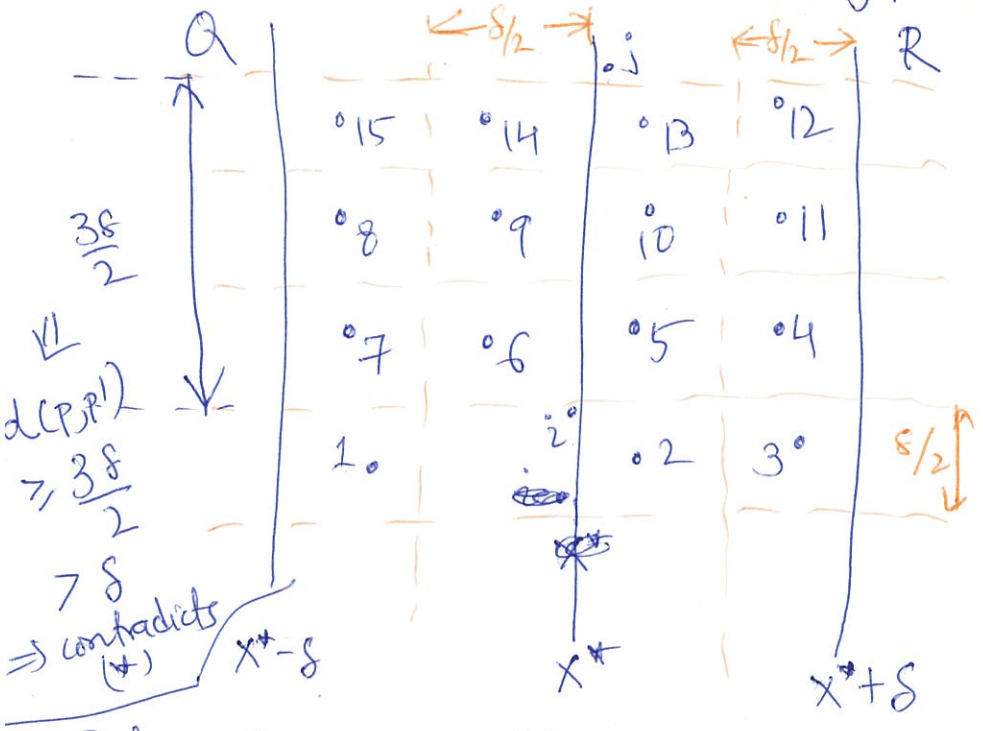
else

return null

$O(|S_y|) = O(n)$

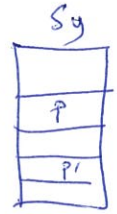
Pf (idea) of Kickass Property lemma

For contradiction assume $\exists |t-j| \geq 16$



$\Rightarrow$ contradicts (*)

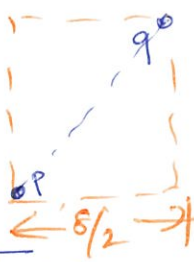
But each square inside Q or R $\Rightarrow$ contradicts the definition of δ ☐



$d(p,p') < \delta$ ——— (*)

Claim: Every $\delta/2 \times \delta/2$ squares has ≤ 1 point from S in it.

Pf (idea): Assume p & q in square



Ex: p & q are furthest if placed on diagonal corner

$$\Rightarrow d(p,q) = \sqrt{\left(\frac{\delta}{2}\right)^2 + \left(\frac{\delta}{2}\right)^2} = \sqrt{\frac{\delta^2}{4} + \frac{\delta^2}{4}}$$

$$= \sqrt{\frac{\delta^2}{2}} = \frac{\delta}{\sqrt{2}} < \delta.$$