Nov 26

Simplified problem: Only compute value of an optimal solution / schedule

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$OPT(j)$ = value of optimal solution $[j]$ $(0 \le j \le n)$

$[j] = \{(s_1, f_1, v_1), \ldots, (s_j, f_j, v_j)\}$

Goal: Compute $OPT(n)$

Def: O_j to be an optimal solution for $[j]$

$$OPT(j) = v(O_j)$$

Goal: Recurrence relation:

$$\boxed{OPT(j) = \max\{OPT(j-1),\ v_j + OPT(p(j))\}\ ;\ OPT(0) = 0}$$

Case 1: $j \notin O_j$ Claim 1: O_j is also optimal for $[j-1]$

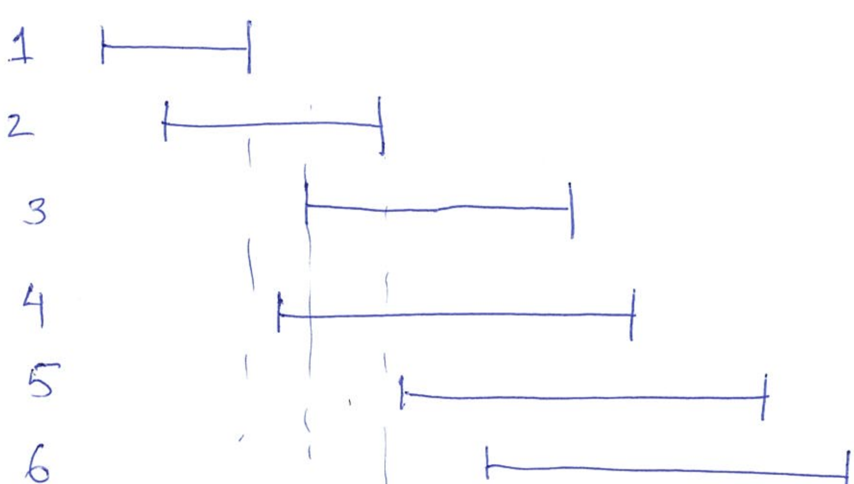
$$\Rightarrow OPT(j) \underset{\text{def of } O_j}{=} v(O_j) \underset{\text{claim 1}}{=} OPT(j-1)$$

Pf (idea) of Claim 1: Assume $\exists$ schedule O' for $[j-1]$ s.t $v(O') > v(O_j)$

BUT O' is also a valid schedule for $[j] \Rightarrow$ contradicts optimality of O_j for $[j]$

Def: $p(j)$ is the largest $i < j$ s.t i & j do not conflict
$= 0$ (if no such i $\exists$)

Ex:



$p(1) = 0$
$p(2) = 0$
$p(3) = 1$
$p(4) = 1$
$p(5) = 2$
$p(6) = 2$

Note:
(i) $p(j)+1, \ldots, j-1, j$ conflict with j
(ii) $1, \ldots, p(j)$ does NOT conflict with j

Case 2: $j \in O_j$ Claim 2 $O_j \setminus \{j\}$ is an optimal solution for $[p(j)]$

$\overset{\text{Claim 2}}{\Longrightarrow}$ $OPT(j) = v_j + OPT(p(j))$

Pf (idea) of Claim 2: Assume O' is a valid schedule for $[p(j)]$

s.t. $v(O') > v(O_j \setminus \{j\})$

Note: $O' \cup \{j\}$ is a valid schedule for $[j]$

but $v(O' \cup \{j\}) = v(O') + v_j > v(O_j \setminus \{j\}) + v_j$
$= v(O_j)$

$\Rightarrow$ contradicts the optimality of O_j for $[j]$ ■

Ex: Compute $p(j)$ for all $1 \le j \le n$ is $O(n \log n)$

Bonus Ex: Computing all $p(j)$ values $\Omega(n \log n)$ comparisons