Nov 28

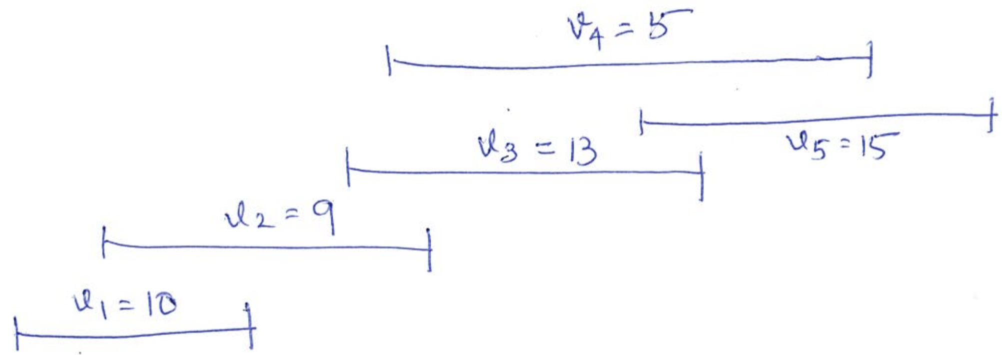
$M[0] = 0$ $\qquad$ $M[0 .. n+1]$ $\qquad$ $(f_1 \le f_2 \le \ldots \le f_n)$

for $j = 1 \ldots n$

$M[j] = \max\{\underbrace{v_j + M[p(j)]}_{j \in O_j},\ \underbrace{M[j-1]}_{j \notin O_j}\}$

return $M[n]$

n=5



~~p(5)~~ $p(4) = 1$

$p(5) = 2$

$p(3) = 1$

$p(2) = 0$

$p(1) = 0$

	0	1	2	3	4	5
j=0	0					
j=1	0	10				
j=2	0	10	10			
j=3	0	10	10	23		
j=4	0	10	10	23	23	
j=5	0	10	10	23	23	25

(j=2: $j-1$, $p(j)$)

$M[0] = 0$

$M[1] = \max\{v_1 + M[0], M[0]\} = \max\{10+0, 0\} = 10$

$M[2] = \max\{v_2 + M[0], M[1]\} = \max\{9+0, 10\} = 10$

$M[3] = \max\{v_3 + M[1], M[2]\} = \max\{13+10, 10\} = 23$

$M[4] = \max\{v_4 + M[1], M[3]\} = \max\{5+10, 23\} = 23$

$M[5] = \max\{v_5 + M[2], M[4]\} = \max\{15+10, 23\} = 25$

Compute an optimal solution:

$[\{5, 1\}]$

$5 \overset{?}{\in} O_5$ $\qquad$ [$O_j \to$ is an optimal solution $[j]$ $\quad OPT(j) = v(O_j)$]

$25 > 23 \Rightarrow 5 \in O_5$

Now consider $O_5 \setminus \{5\} = O_2 \subseteq [2]$

$2 \overset{?}{\in} O_2 \qquad 9 < 10 \Rightarrow 2 \notin O_2$

Now consider $O_1 = O_2 \Rightarrow 1 \overset{?}{\in} O_1 \qquad 10+0 > 0$

$\Rightarrow 1 \in O_1$

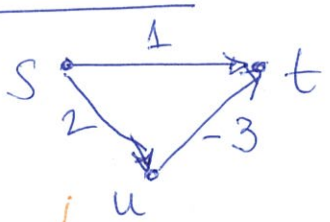
M_schedule$(n; M, p)$
  If $n = 0$ return $\emptyset$
  If $v_n + M[p(n)] > M[n-1] \to$ return $\{n\} \cup$ M_schedule$(p(n); M, p)$
  else return M-schedule$(n-1; M, p)$

Shortest path problem

Input: (•) Directed graph $G = (V, E)$, $\forall e \in E$, cost c_e
(but c_e can be < 0)
BUT no negative cycle
(•) $t \in V$

Output: $\forall s \in V$, output a shortest s-t path
$(c(P) = \sum_{e \in P} c_e)$

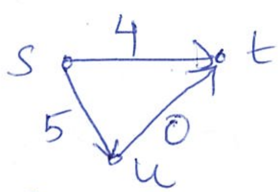
ATTEMPT 1: Run Dijkstra for each $s \in V$



→ start Dijkstra at s
it'll pick s, t as the shortest s-t path
BUT actual shortest s-t path
s, u, t

ATTEMPT 2: Add a large enough +ve number to all edge costs & run Dijkstra on the new instance.

3 to all edgecost

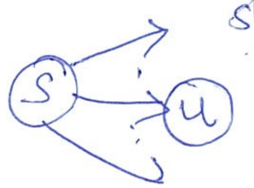


Shortest path s, t

→ No known Greedy / D&C algo

Assume: we are interested in <u>cost</u> of shortest path

ATTEMPT 3: $OPT(s)$ = cost of shortest s-t path.



shortest s-t path takes edge (s,u)

$$OPT(s) = c_{(s,u)} + OPT(u)$$

$$OPT(s) = \min_{u : (s,u) \in E} \{ c_{(s,u)} + OPT(u) \}$$

1. Are there poly many subproblems
2. Is there a recursion?
3. Is there an ~~or~~ ordering among sub-problems?