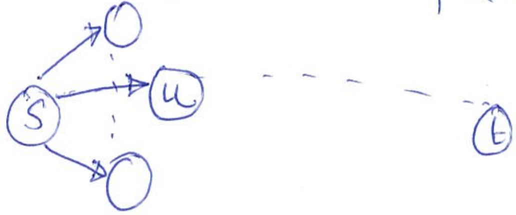
Nov 30

ASSUME: Only interested in cost of shortest s-t paths ($\forall s$)

ATTEMPT 3: Use "weighted interval scheduling like" dynamic prog.

$OPT(s)$ = cost of a shortest s-t path



If a shortest s-t path has (s,u) as its first edge

$OPT(s) = c_{(s,u)} + OPT(u)$

In general

$$OPT(s) = \min_{u:\,(s,u)\in E} \{ c_{(s,u)} + OPT(u) \}$$

Q: Which properties are satisfied

1. Poly many subproblems → n subproblems ✓
2. recurrence formula
3. Ordering among sub-problems.

$OPT(s) = 2 + OPT(u)$

$OPT(u) = \min \{3 + OPT(t), -1 + OPT(s)\}$

Issue: $OPT(s)$ depends on $OPT(u)$ } → no hope of total order.
$OPT(u)$ ——— $OPT(s)$

ATTEMPT 4: $OPT(s, E')$: cost of shortest s-t path in $G' = (V, E')$

$E' \subseteq E$

- $OPT(s, E) = c_{(s,u)} + OPT(u, E \setminus \{(s,u)\})$

more generally: $OPT(s, E) = \min_{u:\,(s,u)\in E} \{ c_{(s,u)} + OPT(u, E\setminus\{(s,u)\}) \}$

Q: Which properties are satisfied?

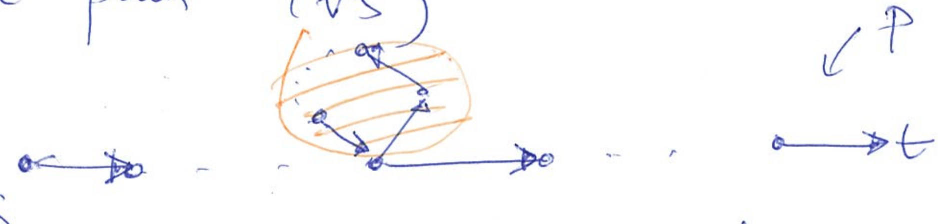
1. Poly many in n sub-problems → $n \cdot 2^m$
2. Recurrence formula
3. Ordering among sub-problems → order according to $|E|$

ATTEMPT 5: (Bellman-Ford Algo)

$OPT(s, i)$ = cost of shortest s-t path with $\leq i$ edges.

PROP: If G has no -ve cycle $\Rightarrow \exists$ a simple shortest s-t path ($\forall s$)

Pf (idea):



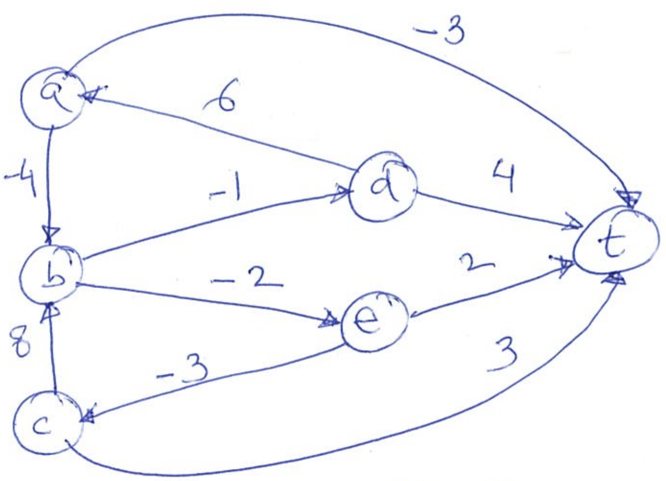
If $\exists$ a cycle T, consider $P \setminus T$ ⇐ still an s-t path

$$c(P \setminus T) = c(P) - c(T)$$
$$\leq c(P) \quad \text{as } c(T) \geq 0$$

$\Rightarrow P \setminus T$ is also optimal (since P is a shortest s-t path)

$OPT(s, i)$ = cost of a shortest s-t path with $\leq i$ edges. ← $0 \leq i \leq n-1$

Q: $OPT(s, n-1) \rightarrow$ final output → By PROP, $\exists$ a simple shortest s-t path. Any simple path $\leq n-1$ edges in it.



Look at d.

$OPT(d, 0) = \infty$ [as $d \neq t$]

$OPT(d, 1) = 4$ [d, t]

$OPT(d, 2) = 6 - 3 = 3$ [d, a, t]

$OPT(d, 3) = 3$ [d, a, t]

$OPT(d, 4) = 6 - 4 - 2 + 2 = 2$ [d, a, b, e, t]

$OPT(d, 5) = 6 - 4 - 2 - 3 + 3 = 0$ [d, a, b, e, c, t]

$\underbrace{OPT(d, 6) = 0 = OPT(d, 7) = \cdots}$

By PROP: always $\exists$ a shortest $\leq n-1 = 5$ edges.