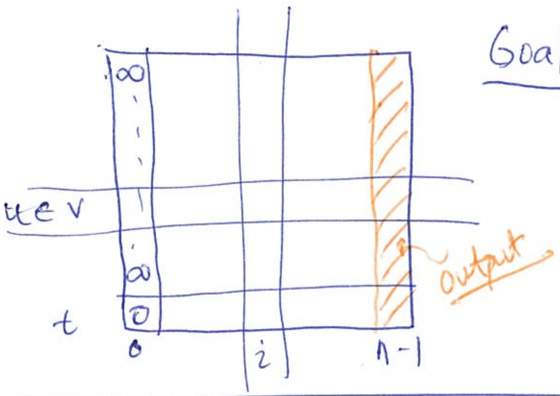
Dec 3: $OPT(u,i)$ = cost of shortest u-t path with $\le i$ edges

$\forall u \in V$, $i = 0 \ldots n-1$



Goal: $M[u][i] = OPT(u,i)$

→ # subproblems $= n^2$ ← poly many sub-problems.

→ Output: $OPT(u, n-1)$ $\forall u \in V$

Recursive formula: $OPT(t,0) = 0$

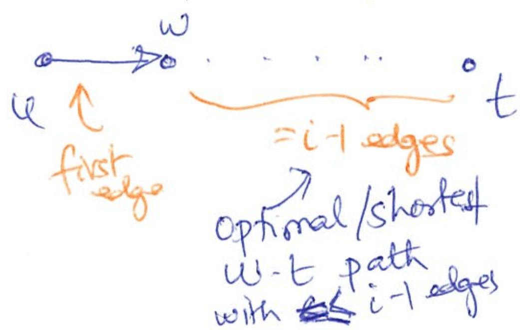
$OPT(u,0) = \infty \quad \forall u \neq t$

$OPT(u,i)$

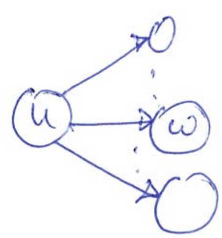
$i > 0$ Case 1: A shortest u-t path with $\le i$ edges actually uses $\le i-1$ edges.

$$OPT(u,i) = OPT(u,i-1)$$

Case 2: All shortest u-t path with $\le i$ edges uses EXACTLY i edges.



$$OPT(u,i) = c_{(u,w)} + OPT(w, i-1)$$

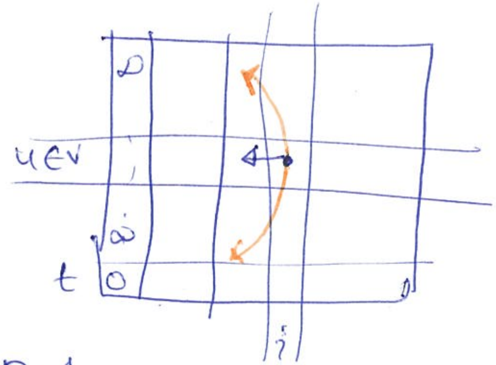


$\cancel{OPT(u,i)} = \min_{w:\,(u,w)\in E}$

$$OPT(u,i) = \min_{w:\,(u,w)\in E} \{ c_{(u,w)} + OPT(w, i-1) \}$$

OVERALL: $OPT(u,i) = \min \{ OPT(u,i-1), \min_{w:\,(u,w)\in E} \{ c_{(u,w)} + OPT(w,i-1) \} \}$

Ordering:



The i^{th} column only depends on column $i-1$

$\rightarrow$ good ordering: column by column (L to R)

Bellman-Ford:

0. Allocate an $n \times n$ matrix M
1. $M[t][0] = 0$, $M[u][0] = \infty \quad \forall u \neq t$
2. For $i = 1..n$

   For $u \in V$

   $M[u][i] = \min \{ M[u][i-1], \min\limits_{w : (u,w) \in E} \{ c_{(u,w)} + M[w][i-1] \} \}$
3. return $M[s][n-1] \quad \forall s \in V$