Gale - Shapley Algo

⓪ Initially all men and women are <u>free</u>

① In a loop:

A free <u>woman</u> proposes to some man.

↑ in book men propose

② You have n matched pairs

<u>Initial state</u>: All n men and women are free.

① Let <u>w</u> be a free woman.

Q: Who should w propose to?

A: the man <u>m</u> on top of her preference list

② Q: What should m do?

Case 2.1: m accepts w's proposal

<u>Issue</u>: m could get a better proposal later

Case 2.2: m rejects w's proposal

<u>Issue</u>: m might not get a better proposal

Case 2.3: m conditionally accepts

⟹ (m, w) are <s>engaged</s> engaged

<u>General state</u>: All men / women are <u>free</u> or <u>engaged</u>

① All n men and women engaged

→ Algo terminates: match up the n engaged pairs

ELSE

② ∃ a free woman w

Q1: ~~Wch~~ Which man m should w propose to?

A1: The best man she has NOT proposed to

③ w propose to m

Q2: What should m do?

Case 2.1: m is free ⟹ (m, w) get engaged

Case 2.1: (m, w') are engaged

Case 2.1.1: w' > w in Lm: no change

Case 2.1.2: w > w' in Lm: (m, w) get engaged, w' is free.

THEOREM: For every input (M, W, 2n preference lists) the S.S algo outputs a stable matching

COROLLARY: Every input of Stable matching problem has a stable matching.