Sep 14 Aside: While USUALLY we will use asymptotic analysis to talk about runtime of an algo, their definitions are INDEPENDENT of the semantics of $g(N)$.

Ex: $g(N)$ = # times Kiran says no to me in the nth month of existence.

$T_A(N)$ = max # steps A takes on ANY input of size N.

algo → A; input size → N

→ Talk about $O(\cdot)$ & $\Omega(\cdot)$ of $T_A(N)$

Example: Search problem: i/p: $a_0, \dots, a_{n-1}; v$

o/p: i s.t. $a_i = v$ (if $\exists i$)

-1 o/w.

$N = n+1$

SEARCH$(a_0, \dots, a_{n-1}; v)$

for $i = 0 \dots n-1$ ← T_1: # iteration of this loop

if $a_i == v$

return i } T_2: time for body of loop

return -1 ← T_3: time taken for this statement.

Claim1: $T_{SEARCH}(N)$ is $O(N)$

$T_1 \le n$

$T_2 \le O(1)$

$T_3 \le O(1)$

$$T_{SEARCH}(N) \le T_1 \cdot T_2 + T_3$$

$$\le \underbrace{n \cdot O(1)}_{\text{product}} + O(1) \le O(n)$$

$$\le O(n) + O(1) \le O(n) + O(n) \underset{\text{additive}}{\le} O(n) = O(N)$$ ■

Claim2: $T_{SEARCH}(N)$ is $\Omega(N)$. ($\Rightarrow$ is $\Theta(N)$)

Recall $T(N)$ is max # steps taken by any i/p of size N

= max { ______ 1st i/p ______,

______ 2nd i/p ______

⋮ }

Pf idea 1: ① Figure out worst-case input of size N ② Analyze runtime on that input.

Pf idea 2: Exhibit one input of size N (for all large enough N) where algo takes $\geq L$ steps $\Rightarrow T(N) \geq L$ (in our case $L = \Omega(N)$)

Pf details: Fix any $n \geq 1$. Consider $a_i = i$ $0 < i < n$, $v = n$

$T(N) \geq \underset{\overset{\shortparallel}{n}}{T_1} \cdot \underset{\geq 1}{T_2} \geq n \cdot 1$ is $\Omega(N) = (\Omega(N))$ ■

"Best case analysis" $a_0 = v \Rightarrow T(N) \geq \Omega(1)$.

Implementing GS algo:

$\boxed{\text{Initialization}} \leftarrow T_0$

while (..) $\leftarrow$ #itr $= T_1 \leq n^2$

$\boxed{\text{Body}} \leftarrow T_2$ (for each itr)

$\boxed{\text{Output } S} \leftarrow T_3$

IF we can ~~ave gue~~ argue $T_0, T_3 \leq O(n)$ $\boxed{T_2 \leq O(1)}$ (?)

$$
\begin{aligned}
T(N) &\leq T_0 + T_1 \cdot T_2 + T_3 \\
&\leq O(n) + n^2 \cdot O(1) + O(n) \\
\text{additive} &\leq O(n) + n^2 \cdot O(1) \underset{\uparrow \text{mult.}}{\leq} O(n) + O(n^2) \leq O(n^2)
\end{aligned}
$$ ■