NAME: ______________________

CSE 331

Introduction to Algorithm Analysis and Design
Sample Mid-term Exam-I: Fall 2019

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DIRECTIONS:

- Closed Book, Closed Notes except for one $8\frac{1}{2}$" $\times$ 11" review sheet.
- Time Limit: 50 minutes.
- Answer the problems on the exam paper.
- Make sure you write your NAME on the paper.
- If you need extra space use the back of a page.

1a	/5
1b	/5
1c	/5
1d	/5
1e	/5
1f	/5
1g	/5
1h	/5
Total	/40

FEW GENTLE REMINDERS:

- You can quote any result that we covered in class or any problem that was there in a homework or recitation (but remember to explicitly state where you are quoting a result from).
- If you get stuck on some problem for a long time, move on to the next one.
- The ordering of the problems is somewhat related to their relative difficulty. However, the order might be different for you!
- You might be better off by first reading all questions and answering them in the order of what you think is the easiest to the hardest problem. Keep the points distribution in mind when deciding how much time to spend on each problem.

1. ($8 \times 5 = 40$ points) Each of the questions below have two parts. For the first part, you need to give a justification for the statement and is worth 2 points. For the second part, answer True or False and briefly JUSTIFY your answer. A correct answer with no or totally incorrect justification will get you 1 out of the total 3 points. **An incorrect answer *irrespective* of the justification will get you 0 out of 3 points**. You *can* assume part 1 when answering part 2 but to get credit for part 1, you *have* to answer part 1. (Recall that a statement is true only if it is logically true in all cases while it is is false if it is not true in some case).

   (a) Consider an arbitrary instance of the stable marriage problem with n men and n women.

   (**Part 1**) Argue why the following statement is **TRUE**.
   There are $n! = n \times (n-1) \times \ldots \times 1$ many possible *perfect* matchings.

   (**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.
   There are at most $n!$ *stable* matchings for the instance.

   True **False** (Please **CIRCLE** your answer)

   (b) Let $f(n) = \log \log n$ and $g(n) = 10^{10^{10^{10^{10^{10}}}}}$.

   (**Part 1**) Argue why the following statement is **TRUE**.
   $f(n)$ is $\Omega(g(n))$.

   (**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.
   $f(n)$ is $O(g(n))$.

   True **False** (Please **CIRCLE** your answer)

(c) Let $f(n) = n^n$ and $g(n) = 2^{400n}$.

(**Part 1**) Argue why the following statement is **TRUE**.

$$f(n) = 2^{n \cdot \log_2 n}.$$

(**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.

$f(n)$ is $\Omega(g(n))$.

True **False** (Please **CIRCLE** your answer)

(d) Let $a_1, \ldots, a_n$ be n integers.

(**Part 1**) Argue why the following statement is **TRUE**.

Let $a_i \in \{0, 1\}$ for each $i \in [n]$. Then the n numbers can be sorted in $O(n)$ time.

(**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.

Let $-\sqrt{n} \leq a_i \leq \sqrt{n}$ for every $i \in [n]$. Then the n integers $a_1, \ldots, a_n$ can be sorted in $O(n)$ time.

True **False** (Please **CIRCLE** your answer)

(e) Consider the BFS algorithm with its input graph G in adjacency list format.

(**Part 1**) Argue why the following statement is **TRUE**.

The input size for BFS is $\Theta(n + m)$.

(**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.

BFS is a linear time algorithm. (Recall that an algorithm is a linear time algorithm if it runs in time $O(N)$ on inputs of size N.)

True **False** (Please **CIRCLE** your answer)

(f) For any graph, recall that running the BFS algorithm implicitly computes a BFS tree. (Note: BFS tree is *not* rooted.)

(**Part 1**) Argue why the following statement is **TRUE**.

A BFS tree can be computed (explicitly) in $O(m + n)$ time.

(**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.

Every graph has a a unique BFS tree for it.

True **False** (Please **CIRCLE** your answer)

(g) Recall that a directed graph is strongly connected if and only if every pair of vertices have directed paths from one to the other.

(**Part 1**) Argue why the following statement is **TRUE**.

A directed graph has at most n^2 edges in it.

(**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.

Any directed graph on n vertices with at least $n - 1$ edges is strongly connected.

True **False** (Please **CIRCLE** your answer)

(h) Recall that any graph can be represented in adjacency matrix format.

(**Part 1**) Argue why the following statement is **TRUE**.

Adjacency matrix takes $\Theta(n^2)$ space.

(**Part 2**) Is the following statement true or false? Also remember to briefly JUSTIFY your answer.

There is an $O(n^2)$ time algorithm that for any graph on n vertices given in its adjacency matrix, converts it into its adjacency list representations.

True **False** (Please **CIRCLE** your answer)