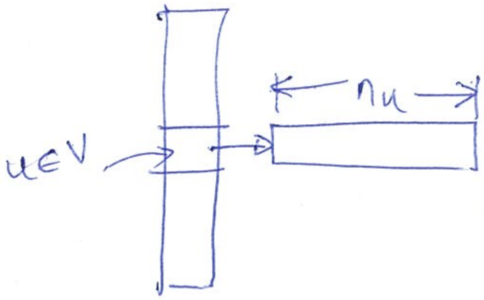
Sep 25

$$\boxed{2m = \sum_{u \in V} n_u}$$

n_u = # neighbors of u = $|\{w \mid (u,w) \in E\}|$
(= degree of u)

Adj. list:



pointers = $|V| = n$
pointers

Sum of lists = $\sum_{u \in V} n_u = 2m$

$\Rightarrow$ Overall space = $n + 2m$ = $\Theta(n+m)$

$\swarrow O(n)$ $\searrow O(n^2)$

all possible pairs

$0 \le m \le \binom{n}{2} = \frac{n(n-1)}{2} \le O(n^2)$

BFS (G, s) // G is in adj. list format

$O(n)$ {
0. $CC[s] = T$ and $CC[u] = F$ $\forall u \neq s \in V$
1. $i = 0$
2. $L_0 = \{s\}$
}

3. While $L_i \neq \phi$ } T_1: # times this loop is run $[T_1 \le T_{123}]$

3.1 $L_{i+1} = \phi$

3.2 for all $u \in L_i$

T_{12}: # times algo gets here $\rightarrow$ for all $(u, w) \in E$

T_{123}: # times algo gets here $\rightarrow$ If $CC[w] == F$

$CC[w] = T$

Add w to L_{i+1} } $\rightarrow O(1)$

3.3 $i{+}{+}$ $O(1)$

4. Return CC } $O(n)$ [pass by value]

Total runtime = $O(n) + \underset{\substack{\uparrow \\ 3.1 + 3.3}}{T_1 \cdot O(1)} + T_{123} \cdot O(1) + O(n)$

$\le O(n) + T_{123} \cdot O(1) + T_{123} \cdot O(1) = O(n) + O(T_{123}$

Goal: Bound $|T_{123}|$.

Analysis 1: $T_{123} = O(n^3)$ $\Rightarrow$ Overall: $O(n) + O(n^3) = O(n^3)$

Analysis 2: $T_{123} \leq n^2$ (Obs: every vertex u appears in ≤ 1 L_i)

Claim: $T_{12} \leq n$ ($\Rightarrow T_{123} \leq n \cdot T_{12} = n \cdot n = n^2$)